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Students' Understanding of the Concepts of Vector Components and Vector Products

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Abstract

In this article we investigate students' understanding of: 1) vector components and, 2) vector products. We administered a test to 409 students completing introductory physics courses at a private Mexican university. In the first part, based on the work of Van Deventer [1], we analyze the understanding of components of a vector. We used multiple choice questions asking for students reasoning to elaborate on the misconceptions and difficulties of graphical representation of the *x*- and *y*-component of a vector. In the rest of this work, we analyze the understanding of the dot and cross products. We designed opened-ended questions to investigate the difficulties on the calculation and the misconceptions in the interpretation of these two products.

Introduction and Objectives

Researchers have studied students' understanding of the vector component [1- 3], the dot product [1-4], and the cross product [1, 3, 4] concepts. We present results of a study that contributes on further understanding of the difficulties and misconceptions on these three concepts.
This article covers three objectives: to analyze **1)** the misconceptions about vector component concepts, **2)** the misconceptions and difficulties with the dot product, **3)** the misconceptions and difficulties with the cross product

Methodology

- This research was conducted in a large private Mexican university.
- Questions were administered to 409 students in their last of three calculus-based introductory physics courses at this institution.
- Figures 1, 2 and 3 show the questions used in this study.
- We used Questions 1 and 2 which are modifications of those designed by Van Deventer [1]. We designed Questions 3-6.
- Population A (half of the participants) answered questions 1, 3 and 4. Population B (other half) answered questions 2, 5 and 6. The selection of these two populations was made randomly.

Results and Discussion: 1. Misconceptions in the Component Concept

Question 1. Which one of the boxes below contains the *x*-component vector of **A**, (i.e., A_x)? Explain your reasoning.
Question 2. Which one of the boxes below contains the *y*-component vector of **A**, (i.e., A_y)? Explain your reasoning.

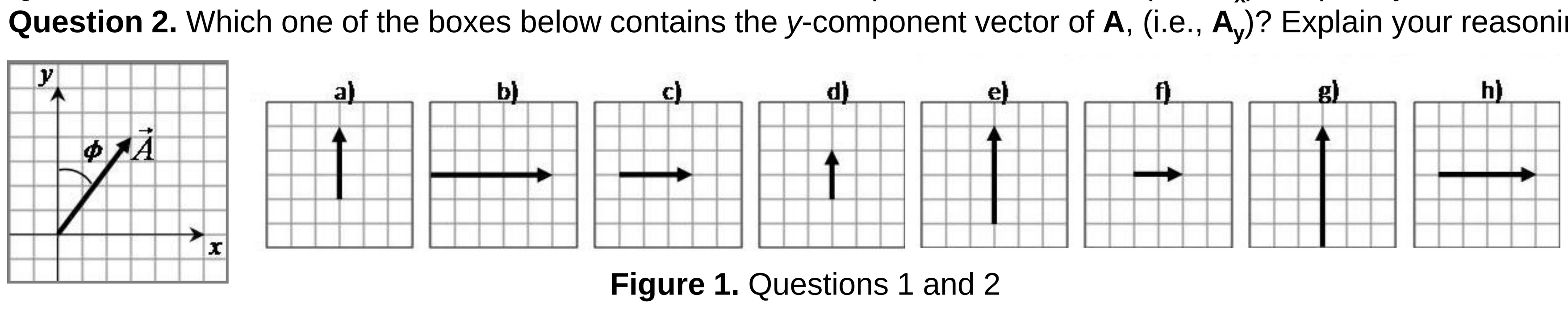


Figure 1. Questions 1 and 2

Table 1. Results of Questions 1 & 2.

Responses	Question 1 *x*-comp.	Question 2 *y*-comp.
a	0%	10%
b	3%	0%
c	87%	5%
d	0%	2%
e	0%	80%
f	4%	0%
g	1%	3%
h	5%	0%

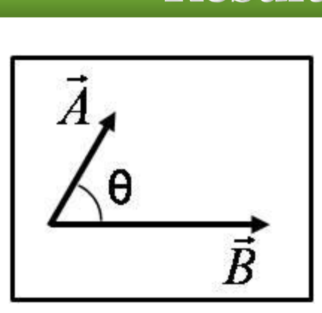
Misconceptions in the *x*-component concept

- The most common error at 8% (options **b** and **h**) is to choose a component with a longer magnitude.
Half of these students state in their reasoning that the magnitude of the *x*-component of vector **A** must have the same magnitude as vector **A**. ***"Because the size of the vector doesn t change, if we move the vector towards the x-axis it would be option b)***."
- 4% choose a component with a shorter magnitude (option **f**). The greater part of these students reasoned as follows: ***"The length of the vector has to be shorter and in the i-direction"***. It seems that these students know the "rule" that the magnitude of a component of a vector must be shorter than the magnitude of the vector. However, they have difficulties to identify the exact graphical representation of the component.

Misconceptions in the *y*-component concept

- The most common error at 12% (options **a** and **d**) is to choose a component with a shorter magnitude. Approximately half of these students justify their answers by stating again the "rule" that the component of a vector has to be shorter than the vector. ***"A component will not have the same magnitude as the resultant vector, it will be smaller, and it will not be half of it, just a little smaller"***.
- 3% choose a longer component (option **g**). Most of these students state in their reasoning that the magnitude of the *y*-component of vector A must have the same magnitude as vector **A**. A student writes: ***"A_y is of the same magnitude as A, even as A_x".*** This student also sketches a circular arc. It seems that he visualizes the components as a vector rotated toward each of the axes.
- 5% choose option **c**, that is, the *x*-component. It seems that there is confusion due to the given angle, i.e., related to the *y*-axis. Some students who select this option relate the component chosen with the "opposite side" in the sine of the angle, which is usually used to calculate the magnitude of the *y*-component when the angle related to the *x*-axis is given.

Results and Discussion: 2. Difficulties with the dot product



Vectors **A** and **B** are shown. The magnitude of vector **A** is 3.0 units, the magnitude of vector **B** is 5.0 units, and the angle θ is 60°.
Question 3. Calculate the dot product (**A•B**) of the two vectors.
Question 4. How can you interpret the dot product (**A•B**) of the last question? (Use the figure from the last question for your reasoning).

Figure 2. Questions 3 and 5

Table 2. Results of Question 3.

Responses	%
Correct	65%
Use equation |**A**||**B**|sinθ	11%
Errors in the components of the vectors	8%
Correct result but with **i**-direction	3%
Multiplication of magnitudes of the vectors	3%
Others	10%

Difficulties in calculating the dot product

- 65% of the students follow the correct procedure. The greater part of these students used directly the equation |**A**||**B**|Cosθ. However, a small part of them added the product of *x*-components and y-components of **A** and **B**.
- The most common error (10%) was to use |**A**||**B**|Sinθ instead.
- Other difficulty students had, when trying to multiply components, is to make errors finding the components of the vectors (7%). The greater part of students with this difficulty had problems with vector **B**. Usually they calculated an *x*-component of **B** using a 60 angle.
- 3% add incorrectly **i**-direction to the correct numerical result, and 3% multiply directly the magnitudes of the vectors.

Table 3. Categories found in Question 4.

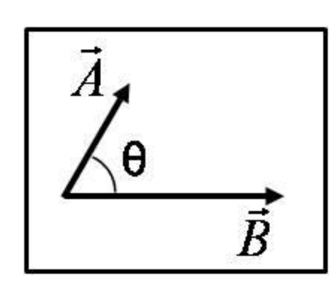
Categories	%
Projection of a vector onto the other vector	11%
Components multiplication	10%
Explaining the equation |**A**||**B**|cosθ	14%
A vector	23%
The magnitude of a vector	9%
No clear explanation	16%
No answer	9%
Others	8%

Misconceptions in the interpretation of the dot product

- 11% relate the dot product with the projection of a vector onto the other vector. These students give the basis of an adequate interpretation. However, only 4% mention explicitly in their rationale that the dot product is the projection of one vector onto a second vector multiplied by the magnitude of the second vector, giving a completely adequate interpretation. Also, 5% interpret the dot product as ***"the projection of one vector onto the other vector"*** only and 2% state in their reasoning that it is this projection plus the magnitude of the other vector.
- 10% relate the dot product with components multiplication. Half of them, instead of talking about the "projection of vector **A** onto **B**", used the term "*x*-component of vector **A**".

The other half explained the equation as $A_xB_x+A_yB_y$. These students, and those who explained the equation |**A**||**B**|cosθ, are only using a definition, which indicates the low level of understanding.
- 23% interpret the dot product as a vector. One third of these students relate the dot product with the sum vector, another third with a bisector vector and a small part with a vector in the positive horizontal direction.
- 9% interpret the dot product as the magnitude of a vector (usually the sum vector or a bisector vector).
- If we add these two errors, we can establish that approximately one third of the population relates the dot product with a vector.

Results and Discussion: 3.Difficulties with the cross product



Vectors **A** and **B** are shown. The magnitude of vector **A** is 3.0 units, the magnitude of vector **B** is 5.0 units, and the angle θ is 60°.
Question 5. Calculate the cross product (**A**×**B**) of the two vectors.
Question 6. How can you interpret the cross product (**A**×**B**) of the last question? (Use the figure from the last question for your reasoning).

Figure 3. Questions 4 and 6

Table 4. Results of Question 5.

Responses	%
Correct magnitude and direction	24%
Correct magnitude and opposite direction	7%
Correct magnitude, no direction given	32%
Use equation |**A**||**B**|cosθ	14%
Errors in the components of the vectors	7%
No answer	4%
Others	12%

Difficulties in calculating the cross product

- 69% of students (the three first responses of Table 4) follow the correct procedure to calculate the magnitude of the cross product. However, 32% of them do not identify the direction of the resultant vector. It seems that these students have problems with distinguishing the difference between the cross product and its magnitude. Furthermore, 31% (first and second responses) of students correctly calculate a vector although 7% of them with an opposite direction.
- 14% incorrectly use |**A**||**B**|cosθ instead of the correct equation.
- 7% make errors finding the components of the vectors. Some students have difficulties with vector **B**; usually they calculate components of **B** using the 60 angle. Others have difficulties with the components of vector **A**. Students usually used the sine function instead of the cosine function (or vice versa).

Table 5. Categories found in Question 6.

Categories	%
A vector, correct direction	22%
A vector, perpendicular to both vectors	8%
A vector, opposite direction	14%
Explain the equation used in the calculation	9%
Sum vector or bisector vector	8%
A "resultant" vector	5%
No answer	7%
No clearly explanation	19%
Others	8%

Misconceptions in the interpretation of the cross product

- Students from the first and the third categories (Table 5) usually sketch vectors in their interpretations. 30% of students (first and second categories) interpret adequately this product in a graphic way and 14% give the basis of an adequate interpretation of this product, but make an error with the direction of this vector.
- 8% interpret the cross product as a sum vector or a bisector vector (usually sketching these vectors), 5% use in their interpretation the term "resultant" vector which does not indicate anything, because it could be that they belong to those who interpret it as a sum or the resultant of a vector product.

Conclusions

- Even after three introductory physics courses, students still have difficulties with vector components, choosing answers with incorrect magnitudes. Some of the students think that the magnitude of a component is equal to the magnitude of the vector, and others know the "rule" that components are shorter than the vector, but have problems to identify the magnitude of the components graphically.
- Students finishing the introductory physics courses have still difficulties to calculate products, related in some part to their problems with components. Also, they have serious difficulties in interpreting the scalar nature of the dot product and the vector nature of the cross product.
- A future study would try to understand whether a physical context would threshold some understanding of these products.

References

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3. R. D. Knight, Phys. Teach. 33(2), 74-78 (1995).
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ACKNOWLEDGMENTS – The authors acknowledge the support received from Tecnologico de Monterrey through grant number CAT140.
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