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Vector Addition: Effect of the Context and Position of the Vectors

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Abstract

In this article we investigate the effect of: 1) the context, and 2) the position of the vectors, on 2D vector addition tasks. We administered a test to 512 students completing introductory physics courses at a private Mexican university. In the first part, we analyze students' responses in three isomorphic problems: displacements, forces, and no physical context. Students were asked to draw two vectors and the vector sum. We analyzed students' procedures detecting the difficulties when drawing the vector addition and proved that the context matters, not only compared to the context-free case but also between the contexts. In the second part, we analyze students' responses with three different arrangements of the sum of two vectors: tail-to-tail, head-to-tail and separated vectors. We compared the frequencies of the errors in the three different positions to deduce students' conceptions in the addition of vectors.

Introduction and Objectives

In recent years, researchers have investigated students' difficulties with the addition of vectors [1-6]; this work contributes further to the understanding of those difficulties.
This work has two objectives: to analyze the effect of **1)** the context and **2)** the position of vectors on two-dimensional vector addition problems.

Methodology

- This research was conducted in a large private Mexican university.
- Questions were administered to 512 students in their last of three calculus-based introductory physics courses at this institution.
- Figures 1 and 4 show the questions used in this study.
- We designed three isomorphic problems (Problems 1-3). Problem 4 was designed by Nguyen and Meltzer [1], and Problems 5 and 6 are our modifications of this problem.
- Population A (one third of the population) solved Problems 1 and 4, Population B (other third) solved Problems 2 and 5, and Population C (other third) solved Problems 3 and 6. The selection of these three populations was made randomly.

Results and Discussion: 1. Effect of the context

Isomorphic problems:
Problem 1. A car travels 3.0 km to the east and subsequently travels 4.0 km to the north. Sketch in the grid the displacement vectors and the total displacement vector.
Problem 2. Two forces are exerted on an object. One force is of 3.0 N to the east and another force is of 4.0 N to the north. Sketch in the grid the force vectors and the total force vector exerted on the object.
Problem 3. There is a vector of 3.0 units to the east and another vector of 4.0 units to the north. Sketch in the grid the two vectors and the vector sum.

FIGURE 1. Problems 1-3

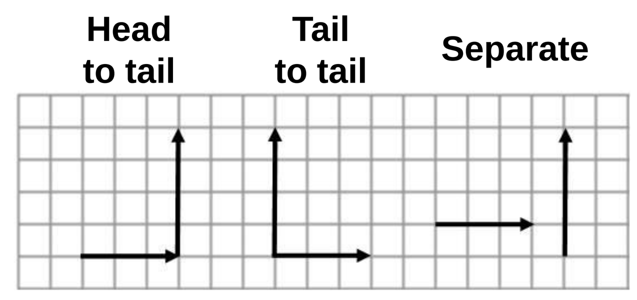

FIGURE 2. Representations by students when sketching the vectors in Probl. 1-3.

TABLE 1. Representations by students when sketching the vectors in Problems 1-3.

Two vectors	Probl.1 Displ.	Probl. 2 Force	Probl.3 No context
Head-to-tail	92%	10%	29%
Tail-to-tail	7%	85%	60%
Separate	1%	1%	6%
Others	0%	4%	5%

Representations by students when sketching the vectors in the isomorphic problems (Problems 1-3).
- In the displacement context, most of students draw the vectors in a head-to-tail representation, and that in the force-context most of them sketch the vectors in a tail-to-tail representation. This is the first difference of these two contexts. These results could be explained by the mental model students have to make to draw the vectors. That mental model in most cases corresponds to the vector context.
- In the no-context problem, students split in two significant percentages; however, the tendency is to draw the vectors in a tail-to-tail representation. It is interesting that in the no-context problem, 6% of students sketch the vectors in the "separate" representation. This is an indication that students have difficulty to make a mental representation of this problem.

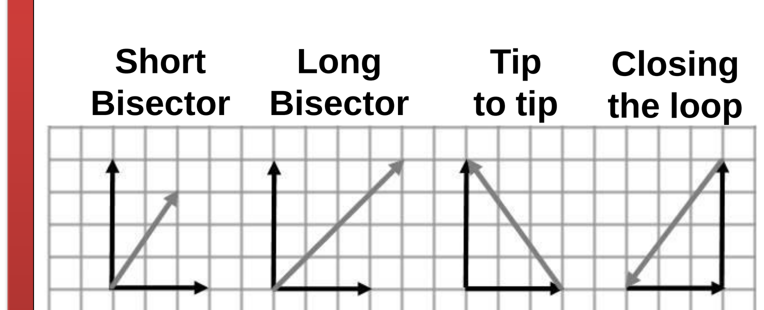

FIGURE 3. Errors in Problems 1-3.

TABLE 2. Vectors sum sketched in Probl. 1-3.

Vector sum	Probl.1 Displ.	Probl.2 Force	Probl.3 No context
Correct	80%	64%	70%
Correct (no dir.)	7%	0%	2%
Short Bisector	2%	17%	10%
Long Bisector	2%	13%	7%
Closing the loop	3%	1%	4%
Tip-to-tip	1%	1%	5%
Others	5%	4%	2%

Differences in the frequencies of the errors in the isomorphic problems (Problems 1-3).
- The displacement-context problem is the one with the most correct answers. This could be due to the fact that this is the most familiar context to students. The most common error is to sketch a vector correct in magnitude but without specifying the direction by an arrow, which could be due to a confusion between displacement and distance or a simple oversight of the student.
- In the force-context and no-context problems, the most common error is to draw a bisector vector, also mentioned by Van Deventer [4]. In this error, students draw a vector sum (with different magnitudes and directions) that goes between the two vectors but lacks the precision to be considered correct. It is possible to distinguish between short bisectors and long bisectors. It is feasible that this error is due to the fact that in the force-context and no-context problems the vector sum is less familiar and more abstract than that in the displacement-context problem.
- In the no-context problem, the errors tip-to-tip and closing the loop are extended. This is another indication that the context influences the answer and that students have difficulties constructing mental models with no-context problems.
- A cross analysis of Table 1 and Table 2 shows that most of the students who draw a bisector vector (short or long) in the force-context and no-context problems, sketch the vectors in a tail-to-tail representation. A small percentage of them draw the vectors in a "separate" representation. This shows an important relationship between drawing the vector in a tail-to-tail representation and the bisector error.

Results and Discussion: 2. Effect of the position of the vectors

Problems 4-6. Vectors **A** and **B** are shown. Sketch in the grid the vector sum **R**, (i.e., **R** = **A** + **B**).

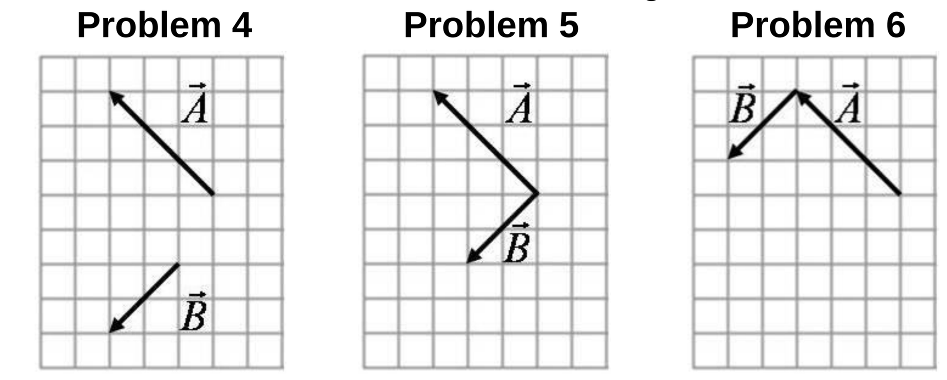

FIGURE 4. Problems 4-6

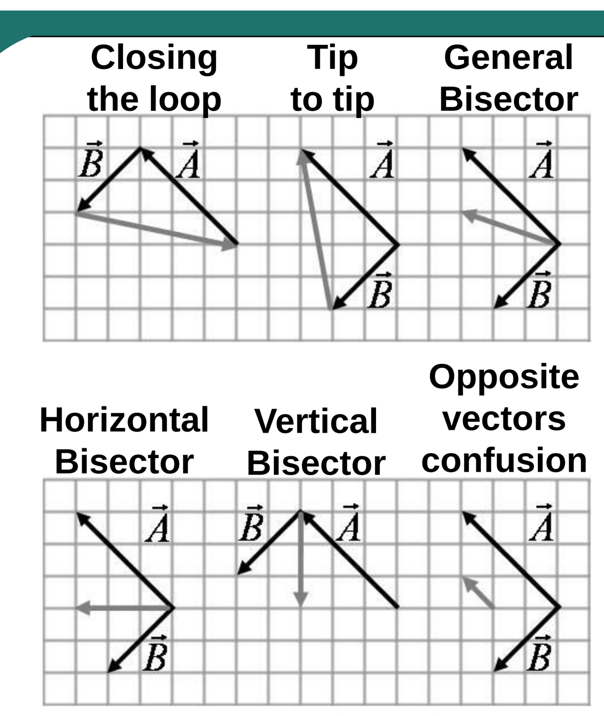

FIGURE 5. Errors in Problems 4-6.

TABLE 3. Vectors sum sketched (Probl. 4-6).

Vector sum	Probl.4 Separate	Probl.5 Tail-to-tail	Probl.6 Head-to-tail
Correct	59%	59%	65%
Correct (no dir.)	2%	1%	3%
Closing the loop	2%	2%	10%
Tip-to-tip	9%	9%	3%
General bisector	6%	8%	4%
Horizontal bisector	9%	12%	5%
Vertical bisector	0%	0%	5%
Opposite vectors confusion	5%	4%	1%
Others	8%	5%	4%

Errors found in Problems 4-6.
- The **general bisector error** is a vector that lacks precision and shows different magnitudes of the *x*- and *y*-components. A student wrote an explanation that exemplified this error: ***"A is greater than B, so R goes more tilted towards A."***
- The **horizontal bisector error** is a vector (with different magnitudes) exactly in the negative *x*-axis. A student wrote a reasoning that exemplified this error: ***"A and B with their directions cancel each other to the center (to the left), and the magnitude is between 3 and 2, that is 2.5".***
- The **vertical bisector error** is a vector that also goes between the two vectors, but the students seem to not realize that it is a head-to-tail representation. This error had not been reported in the literature.
- The **opposite vectors confusion** had not been reported in the literature either. Students usually sketched the vector sum directly, so it is difficult to make a complete analysis of this error. We observed two incorrect procedures that resulted in this particular incorrect answer. **1)** Students use a component addition algorithm subtracting (not adding) the *x*-components. **2)** Students either write this kind of explanations ***"R=3-2"***, ***"R=3A-2B"*** or make sketches that seem to suggest that these students think that the vectors are "opposite", so to add them it is necessary to do a subtraction of them.

Differences in the frequencies of the errors in the three representations (Problems 4-6).
- In the two first representations 59% of students draw the correct sum vector and in the third representation this percentage increases to 65%.
- There are clear tendencies in the tip-to-tip and closing the loop errors. In the two first representations the percentage of the tip-to-tip error is significant (9%), but in the third representation this error is only 3%. On the other hand, the closing the loop error is in the third representation (head-to-tail) significant (10%) and in the other two representations is only 2%.
- The bisector error (general, horizontal and vertical) is important in the three representations. If we compare the frequencies of these three errors, we see interesting tendencies. The general bisector and horizontal bisector error are more common in the second representation, than in the first and, finally, in the third one. On the other hand, the vertical bisector error appears only in the third representation.
- The opposite vectors confusion appears with a significant percentage only in the first and second representations. It seems that these two representations trigger in some way this error.
- In general, we observe that the frequencies of error of the first two representations are very similar. This could be explained by the fact that the first representation is "closer" to the second representation than it is to the third.

Conclusions

- We found important differences in responses of students in three isomorphic vector addition problems. The context has an influence on the representations used when sketching the two vectors needed to be added and on the vector sum. We found that the context matters, not only compared to the context-free case, but also between contexts. The results indicate that the context helps most of the students to make a mental model and then they solve the problem with their own sketch. One can argue the need to teach vectors using a context; however, students should be able to transfer knowledge among different contexts, and that probably is better achieved with a context-free approach. We are currently investigating this.

- We found significant differences between responses to problems in the tail-to-tail and head-to-tail representations, and some similarities between those in the "separate" and the tail-to-tail representations. One can hypothesize that the results would have been different if the separate representation would have been closer to the head-to-tail representation instead. What is interesting is that each representation has its own difficulties and the instructor should be aware to plan the instruction.

References

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