Our best juniors still struggle with Gauss's law: Characterizing their difficulties

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Overview

We discuss student conceptual difficulties with Gauss's law observed in the first semester of upper-division **Electricity and Magnetism** (E&M1).

Evidence from: conceptual diagnostic, exams, and student interviews.

Examples of student difficulties: difficulty with... **the inverse nature** of the problem, articulating complete **symmetry arguments**, and recognizing that in situations without sufficient symmetry it is **impossible (rather than "messy"**) to calculate the electric field using Gauss's law.

How to use Gauss's law

$$\oint E \bullet dA = Q_{enc} / \varepsilon_0$$

To solve for **E**:

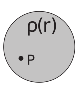
1) Determine from symmetry of charge distribution what direction **E** points and on what variables **E** depends.
2) Create a Gaussian surface on which **E•dA** is know to be either constant or zero
3) Solve for **E** by pulling out of integral

An inverse problem – you can't always solve for **E** using Gauss' law

Quantitative Evidence

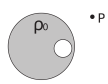
Colorado Upper-Division Electrostatics (CUE) diagnostic [1] – 325 students who have completed E&M1

Do **not** solve, but give "the easiest method you would use to solve the problem" and "why you chose that method"



"… A solid non-conducting sphere, centered on the origin… $\rho(r) = \rho_0 e^{-r^2/a^2}$ Find E (or V) at point P."

33% of students do not recognize Gauss's law as the easiest way to solve



"A charged insulating solid sphere of… ρ_0, with an off-center spherical cavity carved out of it. Find E (or V) at point P, at a distance 4R from the sphere."

24% of students incorrectly choose Gauss's law as the easiest way to solve.

Exam question – 59 students in transformed [2] E&M1

Suppose I evenly fill a cube (length L on a) side with electric charges. I then imagine a larger, closed cubical surface neatly surrounding this cube (length 2L on a side).

A) Is Gauss' law TRUE in this situation? (Briefly, why or why not?) 89% avg. score
B) Can one use Gauss' law to simply compute the value of the electric field at arbitrary points outside the charged cube (Don't try, just tell me if you could, and why/why not?) 46% avg. score
31% of students score 0

Singh [3]: Both introductory and upper- level students do poorly on a Gauss's law diagnostic (49% post-instruction) while graduate students score much higher (75%).

Evidence from Interviews

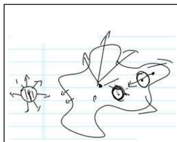
4 students who recently complete E&M1 questions about Gauss's law with a think-aloud protocol.

Inverse Problem

Half of students (who discussed the problem of a spherical Gaussian surface inside an unevenly-shaped insulator of uniform charge density):

- Incorrectly inferred from Gauss's law that **E** at any point on the Gaussian surface was determined only by the charge enclosed.
- Did not clearly distinguish **E** at a single point on the surface and the flux through the surface.

"The E-field… that passes through a Gaussian surface is only dependent on the Q enclosed…On the inside, once again if it's [ρ is] constant, then that's fine, because there's… because it doesn't matter what the shape is looking like 'casue we're not looking on the outside. We're only looking… it's only dependent on the Q enclosed."



A student used Gauss's law to justify that **E** on the surfaces drawn inside the insulator would look the same as drawn to the left around an isolated uniform sphere of charge.

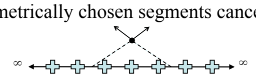
Both students who made these mistakes received above 95% for their overall course score.

Symmetry Arguments

To solve for **E**, use symmetry in step 1)

Experts do this two ways:

- Geometry– charge is invariant with respect to translations/rotations ➔ dependence/direction of **E**.
- Superposition – e.g. horizontal components from symmetrically chosen segments cancel.



Student seem to use superposition only (even when it doesn't work):

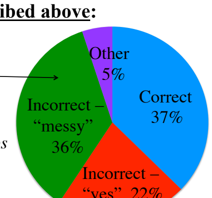
- When asked about **dependence** of **E** near an infinite uniform line charge, ¾ of students use inapplicable superposition argument about **direction** of **E**.
- One student uses superposition argument to decide (incorrectly) that the E-field is uniform near and perpendicular to the external surface of unevenly-shaped insulator of uniform charge.
- No students made a complete argument for both dependence and the direction of **E** – maybe because completely determining both is difficult without employing geometry arguments.

Messy vs. Impossible

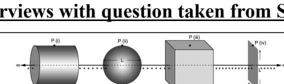
In some situations, students think using Gauss's law to solve for **E** would be difficult, when it is actually impossible:

Exam question part B) described above:

"I don't think so. It probably wouldn't be "simple" because there's no easy symmetry that allows E to be pulled out of the integral, so it'd be mess. Perhaps someone with crazy math skills could."



Our interviews with question taken from Singh [3]:



Which can be used as "Gaussian surfaces to find the electric field magnitude due to the infinite line of charge at a point P…using Gauss's law." Illustration and question from [3, p. 935]

- All students recognize cylinder is easiest.
- All students believe sphere and cube will be difficult but possible:

"I would have to think some more. Maybe do some trig identities… figure it out. It would be a little more complicated [than the cylinder], but we could figure that out."

Results & Conclusions

- A complete understanding of Gauss's law is still lacking for many juniors, including some top students.
- Upper-level students interviewed make incorrect inferences about the electric field based on Gauss's law, do not clearly distinguish between flux and electric field, struggle to articulate complete symmetry arguments, and believe using Gauss's law will be difficult rather than impossible in some situations.
- In upper-level courses it may be helpful to provide **instruction that explicitly addresses these conceptual pitfalls.**

References & Acknowledgements

[1] S. Chasteen, and S. Pollock, "Tapping into Junior's Understanding of E&M: The Colorado Upper-Division Electrostatics (CUE) Diagnostic," in *AIP PERC Proc*. 1179, 2009, pp. 109–112.
[2] S. Chasteen, and S. Pollock, "Transforming Upper-Division Electricity and Magnetism," in *AIP PERC Proc*. 1064, 2008, pp. 91–94.
[3] C. Singh, *Am. J. Phys*. 74, 923–936 (2006).

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