Maximum Likelihood Estimation of students' understanding of vector subtraction

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Likelihood and Probability

Likelihood is not **probability**.

Probability predicts unknown outcomes given known parameters. Conversely, likelihood estimates unknown parameters given known outcomes.

For example, consider a case of flipping a coin 10 times to get 6 heads. The conventional **probability** is the chance of getting 6 heads assuming that the coin bias (chance of getting a single head or tail) is fixed. (The standard hypothesis H_0 is that the coin is fair.) The **likelihood** is the chances of getting six heads as a function of all possible coin biases (from 100% heads to 100% tails).

Our data has an underlying binomial distribution: responses are either leftward or rightward, of magnitude 2 or magnitude 8.

Reading MLE plots

To read this graph, note that the horizontal axis represents the probability of success. The rightmost group of questions have greater likelihoods of probability of success.

For a given observed score, more trials result in shorter, narrower curves. For a given N, more extreme scores also result in narrower curves. This is consistent with our intuition that larger N or extreme score better determine the success parameter, but inconsistent with our expectation that curves involving probability should conserve area.

For each curve, the peak of the curve (the "maximum" in Maximum Likelihood Estimation) corresponds to the observed frequency of success, as is expected in a binomial distribution.

The integral of the likelihood function is not meaningful, and our expectation of conserved area is misplaced.

Context, operation, and alignment

Magntitude

The alignment of the vectors is the best predictor of success in drawing a correct magnitude.

Neither the operation nor the context have much of an effect on magnitude, though the physics context in the opposed condition is statistically different than the others. This is consistent with our intuition that this problem is the "hardest".

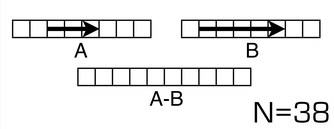
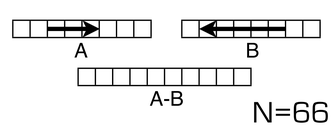
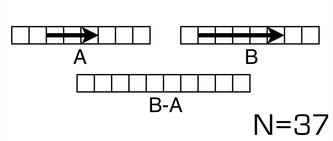
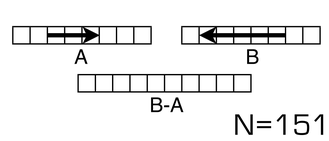
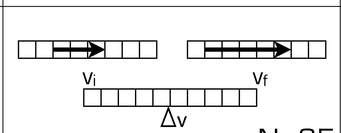
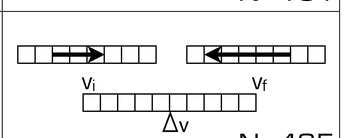
Direction

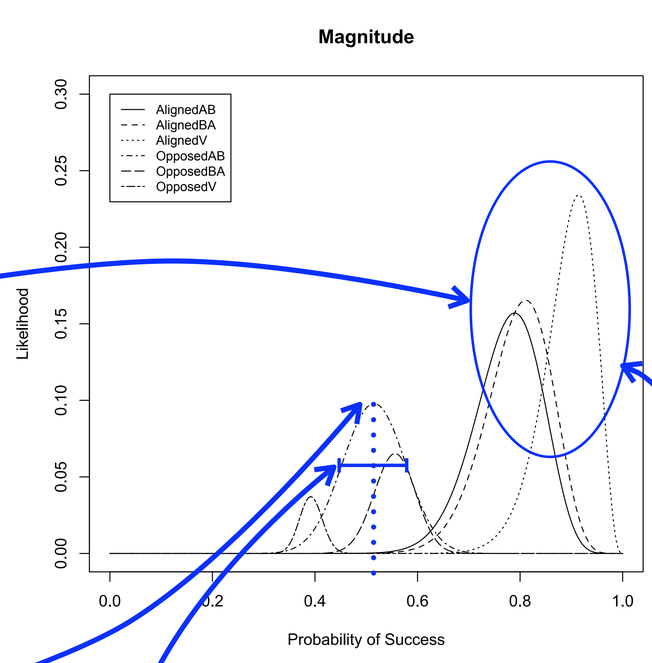
Operation is the best predictor of success, not alignment. Once again, context only significant for the opposed-physics problem.

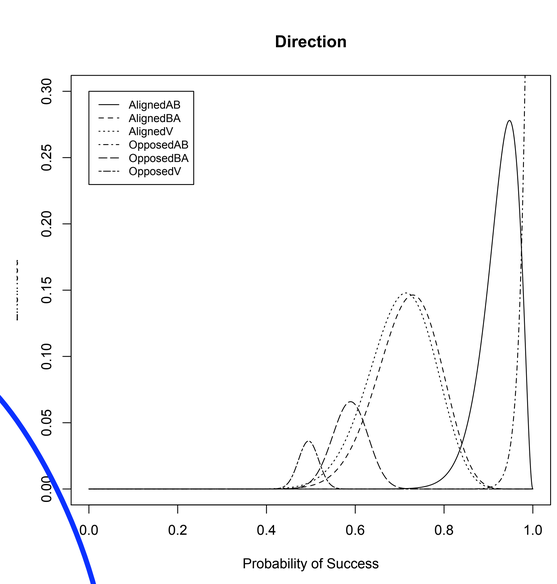
Students in this study are enrolled in a traditional, calculus-based, large, introductory physics sequence for engineers covering Mechanics and E&M. They receive HW credit for effort only.

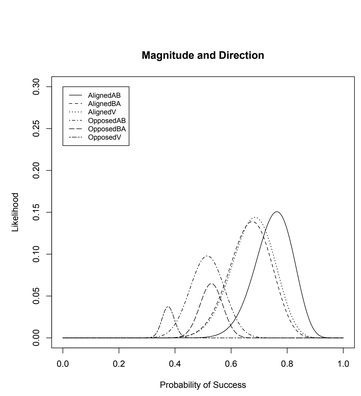
There is no effect of course, instructor, or instructional week on their responses.

Data are between-student within a course; some students enrolled in two courses in succession and may participate twice.

	Aligned	Opposed
[Here] are two vectors, A and B. In the space provided, please draw A-B	 N=38	 N=66
[Here] are two vectors, **A** and **B**. In the space provided, please draw **B-A**.	 N=37	 N=151
[Here] are the initial velocity (v_i) and the final velocity (v_f) for a cart. In the space provided, please draw the change in velocity vector, Δv.	 N=35	 N=485

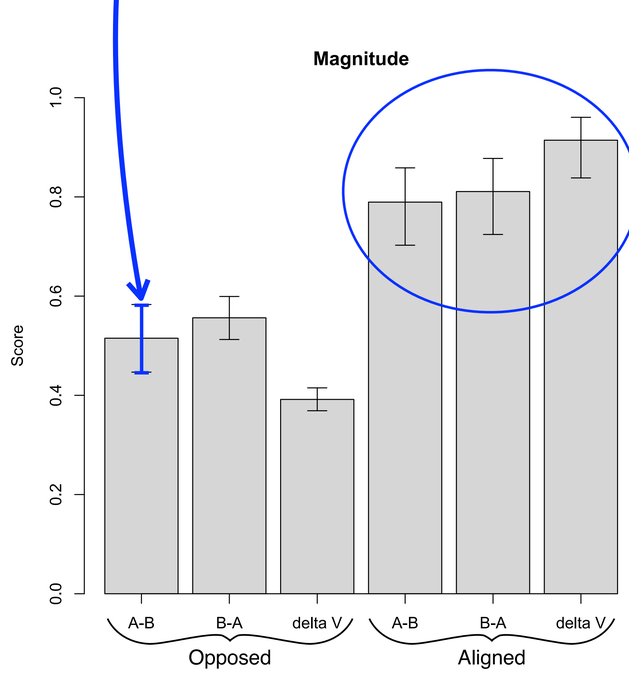


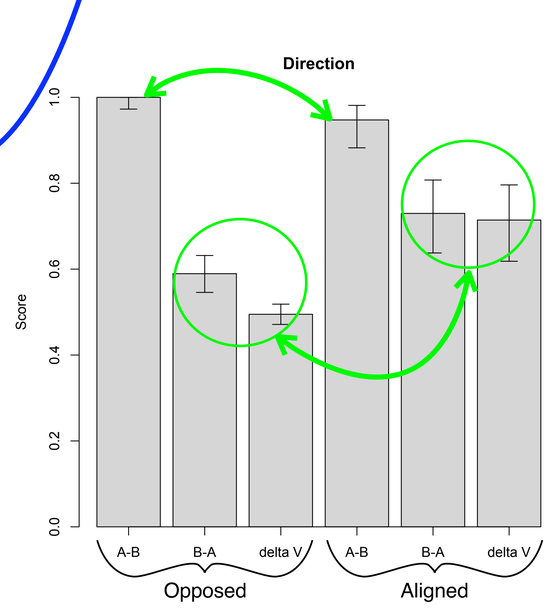


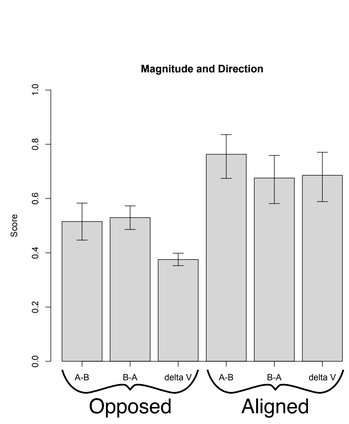


These lines correspond to these bars.

The width of the curve corresponds to the uncertainty in each point. Curves which do not overlap are likely to be statistically different.







RAWR: Rapid Assessment and Web Reporting

RIT and Wabash College are currently developing a web-based system to conduct between student testing. Features include:

- Form subgroups from within a single, large class (e.g. 400 student lecture)
- Automatically distributes weekly assessment to appropriate sub-group

Online reports for instructor including:

- Response-curve and MLE plots comparing groups by task
- Participation by week (for grading)
- Detailed report of all responses per task (for Just In Time Teaching)

RAWR will be alpha-tested during AY 2010-2011. If you are interested in beta-testing RAWR in AY 2011-2012, please contact Eleanor Sayre at LE@zaposa.com.

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