

# Rate of Change and Electric Potential

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**Abstract.** This project aims to investigate a possible underlying cause of student difficulties relating change in electric potential to electric fields. A likely source of difficulties is the lack of students' understanding of the general concept of rate of change (both rate of change in time and in distance). To investigate this link, a diagnostic instrument was created that probed students' understanding of rate of change concepts and electric potential concepts. This paper will report on the creation of the diagnostic instrument and results from student responses.

## INTRODUCTION

More and more colleges and universities are beginning to see the need for reforming introductory physics courses. Evidence for this can be seen in the development and implementation of innovative curricula such as SCALE-UP[1], Studio Physics[2], and Workshop Physics[3]. As developers work to modify the curriculum, they often apply a knowledge of which concepts are poorly understood after traditional instruction. But it may be possible to avoid some of this effort altogether if the underlying causes of the students' difficulties can be identified. If these can be addressed in prior courses, we might see students entering the introductory physics courses without some of the difficulties they now exhibit.

This study focuses on student understanding of the concept of "rate of change" as a possible source for their difficulties with electric potential. The electric field is the gradient of the electric potential. The term "rate of change" has traditionally referred to a derivative of a quantity with respect to time. We have chosen a more general definition that includes both time rate of change and the gradient or "position rate of change". Previous research[4] has shown that students do have difficulties with the concept of rate of change, so it seemed to be a logical place to start because of its underlying importance for understanding electric potential. The topic of electric potential was chosen for several reasons. First,

electric potential is an important idea that later concepts build upon. Knowledge of DC circuits, electromagnetic induction, the photoelectric effect, and quantum mechanics depends upon the students' understanding of electric potential. Because of its fundamental importance, electric potential is one of the earliest topics discussed in the second course in introductory physics. There have already been many studies[5] of student difficulties in kinematics, forces and the other concepts traditionally covered in the first introductory physics course, but research on second-semester topics is limited.

## DESCRIPTION OF RESEARCH

The main approach of this research project was to create a multiple-choice instrument on both rate of change and electric potential concepts (Rate and Potential Test – RAPT) and administer it to a variety of students. The purpose was not to measure the effect of a particular pedagogy or treatment, but rather to investigate the connection between two ideas that a student already holds. There was no need to administer both a pre and a post-test as the study was looking for a link in student understanding rather than looking for a change due to some curriculum. Think-aloud[6] interviews were used to validate the test items and to allow for investigation of any unexpected student responses. The combination of interviews and the

distribution of the multiple-choice instrument allowed us to get a closer look at a few students as well as a more general look at a larger number of students.

The rate of change questions in the instrument ask about the rate of change of a quantity given information about that quantity. For example, there were items that asked about velocity when information about the position of an object was provided. There were not, however, items that focused on the position of an object given its velocity. In mathematical terms, there were questions that called for students to find derivatives, but not carry out integrations. This was done to make the instrument more manageable. If both “derivative” and “integrating” questions were asked, the instrument would be significantly longer. With a long instrument, it is less likely that cooperating instructors would be found. It might also have been less likely for students to take a very lengthy instrument seriously or to complete it without fatigue. In any case, if a connection would be found for “derivative” items, it is likely that there would be a similar relationship for “integration” items as well.

### Development of the Instrument

The questions in the instrument were designed to address several possible modes of thinking students may have about rate of change. These modes were discovered in preliminary interviews. They are: 1) A quantity and its rate of change (like electric potential and electric field) look the same and behave the same. That is, if the quantity increases, the rate of change of that quantity increases. A student using this mode of thinking would say that a graph of the magnitude of the electric field versus position would look the same as a graph of the electric potential versus position. 2) If a quantity is zero, then its rate of change must also be zero. 3) If a quantity is not zero, then the rate of change of that quantity cannot be zero. 4) If a quantity is at its greatest value, then the rate of change of that quantity must also be at its greatest value.

It is important to realize the limitations imposed in the electric potential questions. The goal of these items was to probe students’ understanding of electric potential as it related to the “rate of change” of a quantity. This being the case, the test did not include items that deal with the vector nature of the electric field. The test also was not concerned with the difficulties students have with the differences between positive and negative charges in an electric field.

### Think-Aloud Interviews

As discussed earlier, part of the data collection process included think-aloud interviews of students taking the RAPT. In a think-aloud interview, the student answers the test items while verbalizing the thoughts in his or her mind. Ericsson and Simon[7] describe the general goal of the think-aloud interview as “*procedures to capture what actually goes on in a subject’s mind.*” Of course, the best we can do is approximate what the students are thinking. For this study, we wanted to make sure the students were reading and interpreting the questions as we thought they would. That is, we used the think-aloud interviews partly as a means of validating the questions.

The think-aloud interviews were conducted with eight volunteers from an introductory physics course. The students were paid for their time and the interviews took from 30 to 40 minutes. A multiple-choice question from the FCI was used as a practice question so that students could become familiar with the think-aloud protocol before they answered questions from the RAPT. The practice question also gave the interviewer a chance to comment to the student on how he or she performed with the think-aloud protocols.

### Rate and Potential Scores

The most direct way to examine the relationship between answers on electric potential and rate of change items is to compare scores on these questions. To do this, several sub-scores were calculated. The *rate-score* is the number of rate of change items a student answered correctly, out of a possible 14. The *potential-score* is the number of electric potential items a student answered correctly out of a possible 11. The correlation between these two scores can then be calculated.

Students’ wrong answers also reveal useful information. Do students choose similar incorrect answers for rate of change and electric potential items? To answer this question, two other sub-scores were calculated, the *rate-quantity* score and the *potential-field* score. The *rate-quantity* score is the number of times the students chooses the answer that corresponds to the mental model that says a quantity and its rate of change are directly related. Note that the *rate-quantity* score corresponds to particular responses, thus it is possible for a student to get a 0 rate-score as well as a zero *rate-quantity* score. As an example of the *rate-quantity* score, one item in the RAPT asks about the

acceleration of the coin when it reaches its highest point. The correct answer states that the acceleration is in the negative direction and constant. The *rate-quantity* response for this item is that the acceleration is zero, implying that the acceleration (which is the rate of change of velocity) is zero because the velocity itself happens to be zero when the coin is at its highest point. There are, of course, other responses possible, but these are not associated with the mental model of, “if the quantity is zero (velocity), its rate of change (acceleration) must also be zero”. Similarly, a *potential-field* score is calculated from responses on the electric potential items for student choices that are wrong but using the mental model that electric potential is proportional to the electric field.

## RESULTS AND ANALYSIS

This section will examine the quantitative results from the RAPT. Qualitative data was collected from think-aloud interviews, but that will not be included in this paper.

### Summary Statistics for the RAPT

The first goal of the study was to determine if the RAPT is an appropriate instrument for the task. The most common measure of reliability of an assessment instrument is the use of a reliability coefficient indicating internal consistency. This coefficient was calculated for the RAPT using the Kuder-Richardson[8] formula 20. The KR-20 calculation reported a reliability coefficient of 0.83 with the sample of 340 students. According to Doran’s[9] guidelines for interpreting these coefficients, the reliability of the instrument is “*fairly high, possible for measurement of individuals*”.

### Correlations Between Rate and Potential Scores

The correlation between the *rate-score* and the *potential-score* was found to be 0.45 using the Pearson product-moment coefficient. Fisher’s Z-transformation[10] was then used to determine that this correlation coefficient is significantly different than zero at the 95% confidence level, indicating that there is some correlation between the two variables. The correlation between the *rate-quantity* score and the *potential-field* score was found to be 0.38 which is

also significantly different from zero at the 95% confidence level.

## CONCLUSIONS

It seems clear that students have difficulty with the concept of rate of change as well as the concept of electric potential. The aim of this research was to look for a link between these two difficulties. We have, in fact, uncovered a correlation. Why is evidence of such link important? It is important mainly for curriculum development in the realm of electric potential. In order to write curricula to address student difficulties, developers need to look not only at the concept at hand, but also at supporting concepts.

From the interviews and the multiple-choice data, it seems certain that students make similar mistakes in questions involving rate of change and those involving electric potential. This does not necessarily mean that one causes the other, it is possible there is some other conceptual difficulty that affects a student’s understanding of both the rate of change concept and electric potential. If there is such a underlying cause, it could not be found from the data gathered in this study.

An important “next step” is to create a separate diagnostic instrument solely on the concept of rate of change. With such an instrument two things could be done. First, it could be used to determine what mathematical tools students carry as they enter the introductory physics course. Second, it could be used to investigate the possibility that rate of change difficulties are the source of problems in other areas of physics, such as electromagnetic induction.

This finding also suggests a path for curriculum development. It would seem the best way to approach the concept of electric potential would be to take time to first review the concept of rate of change. Situations where students already understand rates of change, such as the gradient of a hill, can be used as a “bridge” to the concepts the students don’t understand. This is similar to the technique suggested by Clement[11] in which concepts students understand are used to bring students to an understanding of difficult new ideas.

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