

Investigation of Students' Preconceptions and Difficulties with the Vector Direction Concept at a Mexican University

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Abstract. In this paper, we investigate common preconceptions that Mexican university students have with the concept of direction of a vector. Students entering a large private Mexican university were tested before receiving instruction related to vectors in an introductory physics class. In the first part we present students' difficulties with the direction concept of a vector due to the use of two conventions in the Mexican system. The common convention of direction in the American system conflicts with a convention of this property as it is composed of two separate properties: *direction* as the line of action and *sense* as which of the two ways the vector points along that line. Both conventions are regularly used in the Mexican educational system and students use one or the other without doing it explicitly. In the second part, based on the work of Nguyen and Meltzer [1], we designed problems in which students are asked for direction of a vector without indicating any particular convention, and problems in which students are asked for direction of a vector indicating the line of action convention. We analyze preconceptions of direction in the first type of problems (investigating in depth the ones detected by Nguyen and Meltzer), and preconceptions of direction and sense in the second type of problems. At the end we compare responses of students in the two types of problems.

Keywords: Vector, direction, sense, direction convention, preconceptions.

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INTRODUCTION

Students in introductory physics courses have difficulties with the use of vectors [1-3]. In this work we investigate the preconceptions and difficulties in the concept of direction of a vector of Mexican students. There exists a particular difficulty in this concept since in Mexican universities there are in use two different conventions. In the first convention, like in many places in the world, the direction of a vector includes *direction* and *sense*: *direction* is identified as the line of action of the vector (a line parallel to the vector) and *sense* is which of the two ways the vector points along that line. We decided to call this convention as the "line of action convention." Some authors mentioned that in some parts of the world this *direction* (the line of action) is *orientation* [1]; we have indicated elsewhere that in the Mexican educational system this property is also called *direction* not *orientation* [4]. The second convention that Mexican universities use is the "American convention" which is used in the US. The difficulty arises with the use of two conventions, i.e. *direction* with two different meanings. To illustrate the difficulty, if a Mexican student is asked to choose a vector with the same direction as vector **A**, using the American convention, will choose vectors with the same direction in the American way; however, using

the line of action convention, the student will also choose vectors that are at 180° with respect to **A**. Because of this, this type of problems (asking for direction) allows us to know which convention students use.

This study covers three objectives: 1) Illustrate, by showing data, the difficulty of Mexican students due to the use of two conventions. 2) Identify preconceptions in the concept of direction of a vector in problems in which no convention is explicitly used. 3) Identify preconceptions in the concepts of *direction* and *sense* in problems in which the line of action convention is explicitly used.

In the following section we present the details of the methodology of this study. Later, we divided the Results and Discussion section in subsections covering each one of the three objectives. At the end there is a section of Conclusions where we relate the results of the three subsections.

METHODOLOGY

The research was done in a large private Mexican university. There were two parts of the study in consecutive semesters during the academic year 2008/2009. We used as a reference the test from Nguyen and Meltzer [1]. We modified one of the problems to better understand the students'

preconceptions. In the first semester 353 students in an Introduction to Physics (IP) course participated taking our test with open-ended problems before any instruction of vectors. From these results, we designed multiple choice problems which were used in the second semester. The test was administered to 230 students as pre and 199 as post in IP. In this period, interviews were randomly implemented with IP students before and after instruction to inquire more deeply into student understanding [5]. The test was also administered, in this second semester, to 159 students of a calculus-based type course of Electricity and Magnetism (EM) at the end of their course. EM is the last introductory physics course students take in this institution. These students have been using the two conventions without explicitly knowing it. In the two semesters the problems were administered in Spanish.

RESULTS AND DISCUSSION

This section is divided into three subsections where each subsection addresses one of the three objectives above.

1. Difficulty Due to the Use of Two Conventions

In this subsection we show the difficulty with the direction concept due to the use of two conventions. A problem from Nguyen and Meltzer [1] was used. To avoid asking for any of the two conventions, the statement asked only for “direction”. See Fig. 1.

A vector \vec{A} is shown. Next, there is a list of vectors. Which vector(s) does have the same direction as vector \vec{A} ?

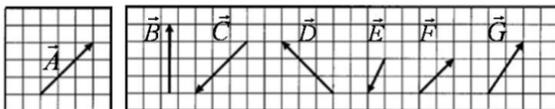


FIGURE 1. Problem 1.

To illustrate the difficulty due to the use of two conventions, let’s compare the results obtained with IP (pre and post) and EM students who use the line of action convention (by choosing **C** and **F**) to those who use the American convention (by choosing **F**). Table 1 shows the results of the second semester. (The table does not include other responses).

Most IP students answer according to the American convention before instruction (these students come from different high schools of the country in which they follow one of the two conventions.) We believe that the language is also a strong factor since direction in Spanish (*dirección*) is more similar to what direction means in the American convention.

TABLE 1. Students’ responses to Problem 1.

Answers-convention used	IP	IP	EM
	pre	post	
C, F (Line of action convention)	4%	2%	15%
F (American convention)	56%	73%	62%

After instruction, the percentage of IP students answering using the American convention increases. During the IP course, a large emphasis is the use of this convention where normally instructors discourage using the word sense. On the other hand, the percentage of EM students using the American convention decreases and using the line of action convention increases. During the courses after IP, no convention is emphasized. This increase and decrease of the use of the American convention indicates a difficulty of having two conventions.

2. Preconceptions with the Concept of Direction

The objective of this subsection is to identify the preconceptions in the concept of direction of a vector in problems in which no convention is asked explicitly. To do this, Problem 1 was used. Since no convention is used, the results could be interesting for instructors teaching in any convention.

At the beginning of this study there was a strong interest to understand the causes of the most common wrong answer reported by Nguyen and Meltzer [1]: vectors **F** and **G**. The authors present two hypotheses: 1. confusion about the requirement that vectors with the same direction be parallel to each other, and 2. confusion about how to recognize when two vectors are parallel.

To investigate these hypotheses, Problem 1 was administered using two versions of the inclination of vector **G**: the original (\mathbf{G}_{orig}) [1] and a new modified inclination (\mathbf{G}_{mod}). See Fig. 2 to note that the inclination of \mathbf{G}_{orig} is very similar to the one of vector **F** and some students could very well confuse whether they are parallel or not. We believe that with \mathbf{G}_{mod} , all students choosing **F** and \mathbf{G}_{mod} , have the confusion of the requirement to be parallel (hypothesis 1).

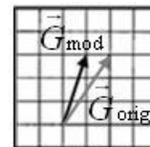


FIGURE 2. The modified version of vector **G**.

Problem 1, in the first semester, was administered to IP students before instruction. In that case the problem was open-ended and students were asked for reasoning. Even with the change in inclination, 12% of students chose **F** and \mathbf{G}_{mod} . Some of these students mentioned explicitly that **A** and \mathbf{G}_{mod} were non parallel. A transcription of a student reasoning says: “*Because, although with different angle, the two vectors (**F** and \mathbf{G}_{mod}) are moving towards the Northeast.*” With this type of reasoning we can conclude that those students have the confusion of the parallelism requirement (hypothesis 1).

Moreover, something very interesting was discovered in the first semester. Another student reasoning is transcribed: “*The direction of vector **A** is towards the Northeast. Only vectors **F** and **G** are in that direction because they are in the first quadrant. Vectors **B**, **C**, **D** and **E** have different directions and different quadrants.*” With this reasoning we can infer that some students not only have the confusion of the parallelism requirement, but also have the preconception that vectors with the same direction must point within a range of angles between north and east in the cardinal system, or point within a range of angles in the first quadrant with the vector “foot” at the origin in a Cartesian system. That is, students relate direction with regions and quadrants. This type of reasoning was also found with the interviews. In addition, although not a high percentage, some students include vector **B** with vectors **F** and **G** as the answer, that is, they include vector **B** “*because is in the limit of the quadrant.*”

In the second semester, to more deeply inquire in this incorrect answer and other preconceptions, this problem was administered with multiple choice answers before instruction. In this semester we wanted to quantify the different preconceptions (hypotheses 1 and 2). Problem 1 (in the original and modified versions) was administered randomly among students. Table 2 presents the results.

TABLE 2. Answers to problem 1. Data from IP pre.

Answers	Original Version	Modified Version
F, G	38%	28%
B, F, G	2%	5%
C, F	4%	5%
F	56%	57%
C, D	0%	0%
None	0%	5%

With these results we can establish that from 38% of students who choose options **F** and \mathbf{G}_{orig} , 28% have problems with the parallelism requirement (hypothesis

1) and may have the idea to relate direction with the region or quadrant. If we add the students who answer option **B**, **F**, \mathbf{G}_{mod} , who probably have the same preconception, the number increases to 33% of students. The other 10% of students (the difference between the 38% and 28%) appears to have a problem recognizing parallel lines (hypothesis 2). The modified version of problem 1 has not been administered in the US. We believe that American students could also have this type of preconceptions.

3. Preconceptions with the Concepts of Direction (Line of Action) and Sense

In this subsection we identify the preconceptions in the concepts of *direction* and *sense* in problems which explicitly the statement uses the line of action convention. The format of Problem 1 was used with the difference that in this problem (Problem 2) the statement asked for a vector with the same “direction and sense” as vector **A**. The results of the first semester show that students have the same preconceptions; however, with different proportions. Therefore, in the second semester, we use the same multiple choice options as the ones for Problem 1. Table 3 presents the results of Problem 2 obtained with IP students before instruction using \mathbf{G}_{mod} .

TABLE 3. Answers to problem 2. Data from IP pre.

Answers	Modified Version
F, G	13%
B, F, G	7%
C, F	3%
F	71%
C, D	3%
None	3%

The result is that 13% of students choose vectors **F** and \mathbf{G}_{mod} , relating *direction* and *sense* with the region or quadrant. 7% of students choosing **B**, **F** and **G** have also this preconception having the “*vector **B** in the limit.*” Note that there are fewer students with this preconception compared to those who responded to the same problem but just indicating direction on the statement (Table 2). This could be caused by asking them for two different properties (*direction* and *sense*) instead of just one (direction). Since from this problem it is not possible to conclude whether students relate *direction* or *sense* (or the combination) to a region or a quadrant, a third problem was designed.

Sense is a concept that depends on *direction* in the line of action convention. Therefore, it was necessary to design a problem with the two concepts but that allows us to know the preconceptions of each concept.

As an option, we designed a problem that asked for a vector with the same *direction* but different *sense*. In the interviews we observed that many students thought it was not possible. Therefore, it was necessary to ask whether it was possible or not to have a vector with the same *direction* and different *sense* as another vector at the beginning of the problem. See Fig. 3.

A vector \vec{A} is shown. a) Is it possible to have a vector that is in the same direction but different sense as vector \vec{A} ?
 b) If your answer is yes, which vector(s) from the list have the same direction and different sense compared to vector \vec{A} ?

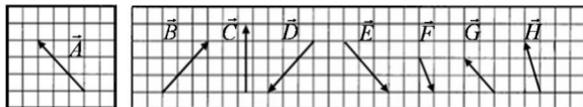


FIGURE 3. Problem 3.

In the second semester IP students solved this problem before instruction. Table 4 presents the results (the percentages are with respect to the total number of students taking this problem).

TABLE 4. Answers to Problem 3. Data from IP pre.

Question a)	%, from total	Question b)	%, from total
Yes	69%	E	36%
No	31%	H	10%
		B, D, E	5%
		D	5%
		B	4%
		G	3%
		G, H	3%
		Others	3%

69% of students answered “yes” to question a), that is, they recognize that there are two different concepts (*direction* and *sense*). Therefore, these students know in some way the line of action convention. 36% (of the total) chose vector **E** following completely and correctly this convention.

The preconceptions could be inferred from the extensive interviews that were implemented with this problem. Table 4 quantifies these preconceptions. 31% of students answered “no” to question a). We can say that many of these students use the American convention answering the question. In the post-test, there were 43% of students who answered “no” (not shown). The IP course emphasizes the American convention, so this is the cause of this increase percentage. However, 35% of EM students answered “no”. In the following courses no clear convention is used by instructors. These results indicate that the answer “no” is related to the American convention.

From students who chose “yes”, those who answered vector **H**, may have the preconception relating *direction* with the region or quadrant and *sense* with the “*exact angle of inclination*.” This would explain the decrease of choosing **F** and \mathbf{G}_{mod} in problem 2.

Students who chose **B**, **D** and **E** may have the preconception relating *sense* with the region or quadrant and *direction* with the angle with respect to the *x*-axis. Those students choosing only **B** or only **D** may have the preconception relating *sense* with greater regions, up-down or right-left respectively. This could be due to the weak understanding of the line of action convention saying “*sense is whether the vector points this way or that way.*”

CONCLUSIONS

In Problem 1 most students answered using the American convention. However, in Problem 3 (question a), more students answered using the line of action convention. This is an apparent contradiction. However, there are many cases in which students respond depending on the question asked, using their own resources [6]. The wording of the problems appears to trigger one of the two conventions. This “contradiction” is evidence of students’ difficulties when they have learned two conventions in a non-explicit way. Future investigation will inquire on this matter. It is also important that in both type of problems, i.e., Problem 1 in which the statement does not use a particular convention or Problems 2 and 3 in which the line of action convention is used, students have preconceptions that relates direction (in the American convention) and *direction* and *sense* (in the line of action convention) with regions and quadrants.

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