

A Repeat Performance? Challenges In Developing Robust Conceptual Understanding in Quantum Mechanics

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Abstract. Ongoing physics education research (PER) at Grand Valley State University (GVSU) is guiding the development of instructional materials for teaching modern physics. Results from this project can help address the question: To what extent should we expect upper-level physics students to be able to apply concepts previously covered in class—even those addressed through PER-based instruction at the advanced level—to different situations? Extensive research at the introductory level has already revealed that such transfer is extremely difficult for beginning students to do on their own. Preliminary results from this project suggest that, even among upper level students, specific conceptual and reasoning difficulties must be addressed explicitly and at multiple instances during instruction.

INTRODUCTION

Physics education research (PER) at the introductory level has revealed that most students lack a working knowledge of basic physics concepts after standard lecture instruction, whether at or beyond the introductory level. [1] Results from this ongoing project yield insight on the following question: After PER-based instruction of a given concept (or concepts) in an initial context, to what extent do upper level students correctly apply that same concept (or concepts) in a different context? This question will be explored in the context of stationary bound state wave functions, partly due to the growing research base in student learning of quantum mechanics, [2-6] and also because the qualitative analysis of such wave functions is possible with careful reasoning about probability density and deBroglie wavelength. [7]

CONTEXT OF INVESTIGATION

The primary context of investigation is the sophomore level modern physics (MP) course at Grand Valley State University (GVSU). The course format consists of three 50-min lectures and a 3-hr lab each week. Most students enrolled (8–14 each year)

pursue majors or minors in physics. The course covers special relativity, photon model of light, wave-particle duality, and basic topics in quantum mechanics.

The author served as either the lab instructor or the instructor of the entire course for the GVSU classes to be discussed. The lab portion of each of those classes consisted of a mixture of traditional labs and small-group inquiry-based activities based on new or existing materials from *Tutorials in Introductory Physics*. [8] Written qualitative questions were administered in class as tutorial pretests and post-tests.

Supplemental data will also be presented from a junior-level course in quantum mechanics (QM) at the University of Washington (UW). Unless otherwise specified, the relevant instruction in that course was delivered in traditional lecture format.

STUDENT DIFFICULTIES WITH BOUND STATE WAVE FUNCTIONS

A research task that has been fruitful in probing student ideas about the qualitative behavior of wave functions is referred to here as the “stepped well” question. The students are asked to consider a particle in the potential shown in Fig. 1. They are told that the

wave function $\psi_n(x)$ corresponds to a highly excited energy state (*i.e.*, has many nodes between $x=0$ and $x=L$). Students are asked to compare regions I and II according to (a) the amplitude of the wave function and (b) the spacing between consecutive nodes of the wave function. The students were required to explain their reasoning and to sketch a qualitatively correct graph of the wave function $\psi_n(x)$.

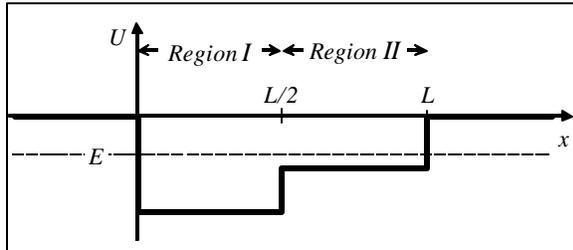


FIGURE 1. Potential energy graph from the “stepped well” question.

Although students did not have the time or requisite math background to solve for the wave function analytically, a qualitative analysis [7] is sufficient. The kinetic energy of the particle is greater in region I than in region II. Semi-classically, the amplitude of the wave function is larger in region II because that is where the particle “spends more time” per unit distance. Also, the (local) deBroglie wavelength is inversely proportional to momentum, so the nodes of the wave function are more tightly spaced in region I, where kinetic energy is greater.

Measuring Conceptual Transfer Of DeBroglie Wavelength and Probability

The “stepped well” question was given to QM students at UW ($N = 61$, Fall 2001) and to MP students at GVSU ($N = 10$, Winter 2002, designated here as “MP class #1”). The UW class had already completed a tutorial on electron interference and deBroglie wavelength [8]. GVSU class #1 had a similar tutorial as well as another tutorial on classical probability.¹ These tutorials have proven effective in enhancing student understanding in the contexts that they explicitly cover. [9, 10]

Even after completing the previous tutorials, however, very few students correctly answered all parts with correct reasoning (see Fig. 2). The low

¹ These tutorials, and others, were pilot-tested at GVSU as part of the FIPSE project P116B000300, “Redefining the Teaching of Applied Quantum Mechanics through the Dissemination of a Proven Reform,” R. N. Steinberg and M. C. Wittmann (2000-2004).

success rates, particularly on the amplitude question, suggest that few students could spontaneously extend their understanding of the requisite ideas to the new context of wave functions. Careful analysis of responses revealed several persistent conceptual difficulties, two of which are discussed below.

Correct responses on “stepped well” question:

(a) In UW QM class (F’02, $N = 61$), after tutorial on electron interference and lectures on 1-D Schr. eqn:		
Amplitudes		(5/61)
Local wavelengths		(24/61)
Both combined		(3/61)
(b) In GVSU MP class #1 (W’02, $N = 10$), after tutorials on electron interference and classical probability:		
Amplitudes		(0/10)
Local wavelengths		(3/10)
Both combined		(0/10)

FIGURE 2. Results from “stepped well” question (a) by QM students at UW and (b) by MP class #1 at GVSU.

Belief That Particle “Spends More Time” In Regions Of Lower Potential Energy

Most students incorrectly believed the amplitude of the wave function would be larger in region I, where the potential energy is lower. (See example of an incorrect student graph in Fig. 3.) Many articulated the idea that the particle, as one student stated, “will want to ‘fall’ into the lower potential.” Students have been shown to hold similar incorrect intuitions about classical situations, such as a block undergoing simple harmonic motion. [10]

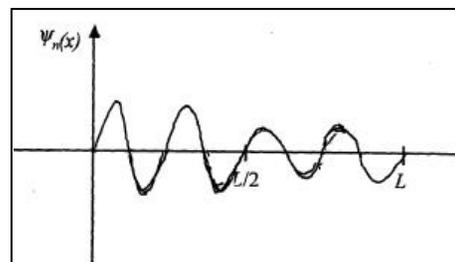


FIGURE 3. Example of incorrect student graph of the wave function on the “stepped well” question. Note the incorrect behavior of both the amplitude and local wavelength.

Failure To Relate DeBroglie Wavelength To Kinetic Energy

Many students also failed to recognize that local deBroglie wavelength varies with kinetic energy. Some associated wavelength with total energy. As one student explained, “the spacing between nodes in region I and II is equal because energy is constant.” The graph in Fig. 3 illustrates this particular error.

ADDRESSING STUDENT DIFFICULTIES: THE NEED FOR MULTIPLE INTERVENTIONS

The results presented above revealed the need to address persistent student difficulties specifically in the context of wave functions. Preliminary versions of new tutorials were tested in the MP classes that took place in Winters 2003 and 2004 at GVSU, referred to here as classes #2 and #3.

Three additional tutorials, described very briefly here, have been tested to date. The first, *Square well potentials (SqW)*,² is designed to help students compare and contrast bound state wave functions for infinite and finite square “well” potentials. It also guides students to recognize that the absolute square of $\psi_n(x)$ yields the classical probability density in the limit of large n , thus motivating the correspondence principle. The next tutorial, *Reflection and transmission of matter (R&T)*,³ focuses on scattering from potential “steps” and tunneling, with particular emphasis on relating local deBroglie wavelength to potential energy. The third tutorial, *Relating classical mechanics to quantum mechanics (RCQ)*,³ helps students apply both local deBroglie wavelength and the correspondence principle in the context of more complicated potentials, including the “stepped well” and harmonic oscillator potentials.

Assessment Of Tutorials Square Well Potentials And Reflection and Transmission of Matter

The “stepped well” question was used as a tutorial post-test after *SqW* in class #2 and after both *R&T* and *SqW* in class #3. The results from these classes are summarized in parts (a) and (b) of Fig. 4. Despite the small class sizes, interesting trends can be noted.

² Initially developed at GVSU.

³ Initially developed at UW and subsequently modified at GVSU.

Correct responses on “stepped well” question:

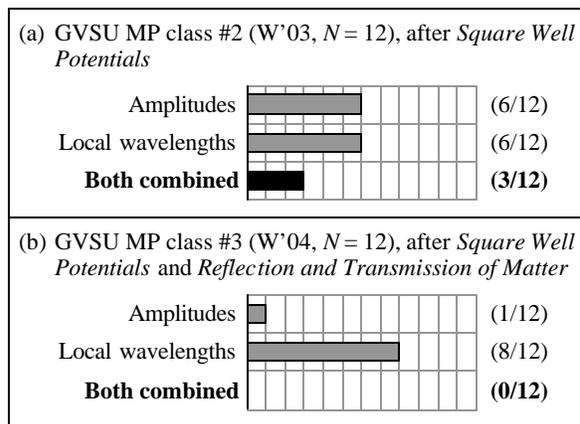


FIGURE 4. Results from GVSU modern physics classes #2 and #3 on the “stepped well” question.

In class #2 half of the students correctly answered on the amplitudes question. The modest level of success could be attributed to the *SqW* tutorial and to its emphasis on the correspondence principle.

Most of the students in class #3 were successful on the wavelengths question, which can be attributed to the *R&T* tutorial. However, very few answered the amplitude question correctly, even though *SqW* was the tutorial completed most recently before administering the “stepped well” question. Many students seemed to make a naïve association between a potential “step” (as discussed in *R&T*) and the “step” inside the stepped “well.” One student explicitly used reasoning regarding reflection and transmission at the “step” to explain why the amplitude would be larger in region I. Others, possibly remembering their experience matching boundary conditions in the scattering problem for the “step,” incorrectly answered instead that the amplitudes were equal in both regions. More in-depth investigation is needed to pinpoint the reasoning underlying these errors. Nevertheless, despite the fact that *SqW* and *R&T* address amplitude and wavelength issues in the context of wave functions, students did not develop a sufficiently robust understanding of those concepts in the context of a more complicated potential “well.”

Assessment Of Tutorial Sequences That Included Relating Classical Mechanics To Quantum Mechanics

The results summarized above motivated the need for the third tutorial (*RCQ*) as well as a new research task, the “unknown potential well” question, to serve as a post-test for this tutorial. On the post-test students were shown a graph of the probability density (see

Fig. 5) for a classical particle in an unknown 1-D potential. Students were first asked to rank locations x_1 , x_2 , and x_3 according to potential energy. They were also asked to sketch the behavior of a highly excited quantum mechanical energy state of the same potential. To answer correctly, students needed to reason “backwards” from the classical probability density graph to potential energy, and then correctly relate local deBroglie wavelength to kinetic energy.

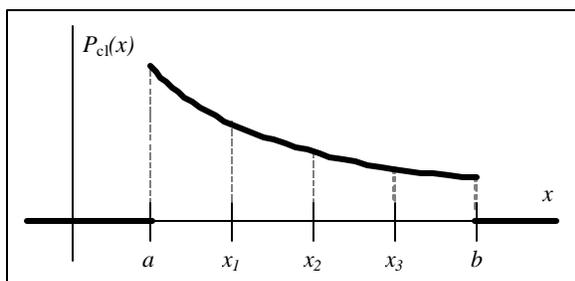


FIGURE 5. Classical probability density graph from the “unknown potential well” post-test, designed for the tutorial *Relating classical mechanics to quantum mechanics*.

The *RCQ* tutorial and the “unknown potential well” post-test (as a final exam question) were used in classes #2 and #3 at GVSU ($N = 24$). The results from both classes were similar even though class #2 did not complete the *R&T* tutorial. The combined results are shown in Fig. 6. About two-thirds of the students answered all parts correctly with correct reasoning.

Correct responses on “unknown well” post-test

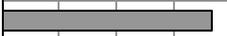
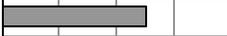
GVSU MP classes #2 and #3 (W'03 and W'04, $N = 24$), after <i>Relating classical mechanics to quantum mechanics</i>	
Amplitudes	 (22/24)
Local wavelengths	 (15/24)
Both combined	 (15/24)

FIGURE 6. Results from GVSU modern physics classes #2 and #3 on “unknown well” post-test. Both classes completed the tutorial *RCQ* as well as one or both of the tutorials *SqW* and *R&T*.

IMPLICATIONS FOR INSTRUCTION AND FUTURE RESEARCH

The promising results from the “unknown well” post-test suggest that the preliminary tutorials on wave functions can enhance student understanding because they provide students with the opportunities necessary to confront and resolve their incorrect ideas and

intuitions. Addressing conceptual difficulties arising in earlier contexts (with deBroglie wavelength in the context of electron interference, and with probability density in the context of classical motion) did not remedy difficulties with those same concepts in the later context of wave functions.

Future research is needed to probe the ability of students to distinguish between the behavior of highly excited and low-lying energy state wave functions, as well as their understanding of probability, probability amplitude, and probability current in both classical and quantum mechanical situations. Nevertheless, the results presented here indicate that ongoing research and curriculum development in upper level courses, despite their typically low enrollments, can yield valuable insight into student learning.

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