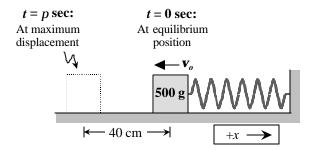1. A 500-g block is placed on a level, frictionless surface and attached to an ideal spring. At $t = 0$ the block moves through the equilibrium position with speed v_o in the $-x$ direction, as shown below. At $t = p$ sec, the block reaches its maximum displacement of 40 cm to the left of equilibrium.

    a. Determine the value of each of the following quantities. Show all work.

        • period: _____

        • spring constant: _____

    b. Using $x(t) = A \cos(wt + f_o)$ as the solution to the differential equation of motion:

        • Determine the form of the function $v(t)$ that represents the *velocity* of the block.

        • Evaluate all constant parameters *(A, w, and f_o)* so as to completely describe both the position and velocity of the block as functions of time.

    c. Consider the following statement about the situation described above.

        "It takes the first π seconds for the block to travel 40 cm, so the initial speed v_o can be found by dividing 40 cm by π seconds."

        Do you agree or disagree with this statement? If so, explain why you agree. If not, explain why you disagree and calculate the initial speed v_o of the block.

2. A simple pendulum consists of a particle of mass *m* suspended by a long, massless wire of length *L.* (*Note:* Neglect all frictional effects.)

    a. Draw a free-body diagram for the pendulum bob corresponding to a moment when the bob is located an angular displacement *f* away from (*e.g.,* to the right of) equilibrium.

    b. Determine an expression (in terms of *m, g,* and *f*) for the component of the net force on the bob that points tangent to the path of the bob.

        • Is this (tangential) component of the net force on the bob characteristic of a *restoring* force? Explain how you can tell.

        • Will this (tangential) component of the net force necessarily result in *simple harmonic motion?* Explain how you can tell.

    c. Now assume that the pendulum bob undergoes ***small*** (angular) displacements from equilibrium. That is, assume that for all values of angular displacement *f* from equilibrium,

        $$\sin f \cong \tan f \cong f \ \ (f \text{ in radians})$$

        i. Simplify your expression for the tangential component of the net force (from part b), and use Newton's Second Law to write down the resulting differential equation of motion for the angular position *f(t)* of the pendulum bob as a function of time.
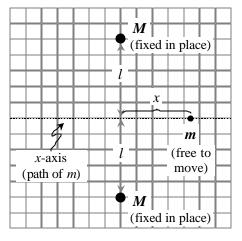
        ii. Explain how your differential equation of motion implies that the pendulum undergoes simple harmonic motion, and determine the frequency of motion in terms of the given parameters. Show all work and explain your reasoning.

3. According to Newton's law of gravitation, the magnitude of the gravitational force exerted on one mass on one mass m_1 by a second mass m_2 is given by:

$$F^{grav} = G\frac{m_1 m_2}{r^2},$$

where r is the distance between m_1 and m_2, and G is the gravitational constant.

Two large solid spheres, each with mass M, are fixed in place a distance $2l$ apart, as shown. A small ball of mass m moves along the x-axis, shown as a dashed line. (Let $x = 0$ represent the point on the axis directly between the spheres, with $+x$ to the right.)

(Figure: Grid showing two masses M (fixed in place) separated vertically, each a distance l from the x-axis (path of m, dashed line). A small mass m (free to move) is located at distance x along the x-axis.)

a. Using Newton's law of gravitation and vector superposition, express the quantities listed below in terms of the given parameters (*i.e.*, in terms of x, l, m, M, and G). Show all work.

  i.   the magnitude of the force on the ball (mass m) by <u>one</u> of the large spheres (mass M)

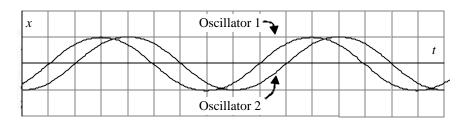 ii. the magnitude of the <u>net</u> force on the ball by <u>both</u> large spheres

b. Use Newton's Second Law, $F_{net} = m\ddot{x}$, to write down a differential equation that governs the motion of the small ball.

c. Is the net force exerted on the small ball a *restoring* force? Explain how you can tell from your differential equation above.

d. Explain why it is *incorrect* to say that the small ball of mass *m* necessarily moves in *simple harmonic motion*. [*Hint:* Contrast your equation of motion in part b of this problem to the equation of motion you solved in the tutorial.]

Under what limiting conditions for *x* is it possible to say that the motion of the mass *m* can be *approximated* as simple harmonic motion? (For large *x?* For small *x?* Large or small compared to *what?*) In this limiting case, determine an expression for the period of motion in terms of the given parameters. Explain your reasoning.

4. Two simple harmonic oscillators (1 and 2) move with equal amplitudes and equal frequencies, as illustrated in the *x vs. t* graph provided.
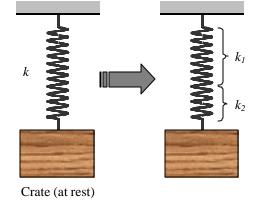

a. What is the phase difference between the motions of oscillator 1 and oscillator 2? (Express your answer as a number of degrees not larger than 180°, or as a number of radians not larger than π.) Explain how you determined your answer.

b. Is oscillator 2 *ahead of* or *behind* oscillator 1 by the amount you specified in part a above? Explain how you can tell.

c. What value of initial phase angle f_o would describe the motion of <u>oscillator 1</u> if you were to express its position *x(t)* as a function of time using:

   • a function of the form $x(t) = A \cos(wt + f_o)$? Explain your reasoning.

   • a function of the form $x(t) = A \sin(wt + f_o)$? Explain your reasoning.

5. An ideal (massless) spring with force constant k is used to hang a crate from the ceiling, as shown.

Suppose that the spring were treated as <u>two separate springs</u> *(1* and *2)* connected end-to-end. Treat each spring as having its own spring constant (k_1 or k_2) given by Hooke's law, $|F_i| = k_i x_i$, where $|F_i|$ is the magnitude of the force exerted on (or by) spring i when the length of that spring changes by an amount x_i.

a. Which choice below correctly relates the magnitudes of the forces F_1 and F_2 exerted by the springs and the magnitude of the force F exerted on the crate? Explain. (*Hint:* Sketch separate free-body diagrams for the springs and the crate.)

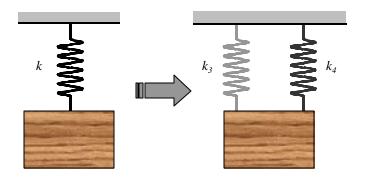(i) $F = F_1 + F_2$ (ii) $F = F_1 = F_2$

b. Which choice below best represents the relationship between the displacements x_1 and x_2 of the individual springs and the displacement x of the original (single) spring? Explain your reasoning.

(i) $x = x_1 + x_2$ (ii) $x = x_1 = x_2$

c. On the basis of your results in parts 1 and 2 above, determine an expression for the spring constant of the original spring *(k)* in terms of the spring constants of the two individual springs. Show all work.

d. Now consider the case in which you replace the original spring with two new springs *(3* and *4)* that connect directly to the crate <u>and</u> to the ceiling. (See diagram.) *Note:* You may assume that each of the new springs has the same natural length as the original spring.

Following the same logic as you did in part a, determine an expression for the spring constant k of the original spring in terms of the spring constants k_3 and k_4 of the new springs. Explain your reasoning.

e. On the basis of your results in parts A and B, if you were to take a spring and cut it in *half,* how would the spring constant for each <u>half</u> of the spring compare to the spring constant of the <u>original</u> (uncut) spring? Explain your reasoning or show all work.