Reasoning Modes, Knowledge Elements and Their Interplay in Optics Problem-Solving

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Abstract. We have investigated how students tackle problems in geometric optics involving ray construction, to try to understand the nature and origin of the surprisingly wide variety of students' solution attempts. We find that students use various reasoning modes and knowledge elements in conjunction. Their thinking may usefully be described as an interplay of principle-based and case-based reasoning, drawing on a knowledge mixture of basic principles, procedures, specific cases and recalled result features. Even though we usually present solutions and teach problem solving as a systematic application of principles, real cognition is more complex. Associative thinking in terms of prior cases seems to be a strong natural tendency of both novices and experts. However, novices are not easily able to discriminate the specific from the general, and tend to lack epistemic awareness and metacognitive skills. Our research findings will be illustrated by examples of student thinking on a basic reflection problem. Implications for learning and instruction are discussed.

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INTRODUCTION

We have undertaken a cognitive study of problem solving in geometrical optics, investigating how students tackle reflection and refraction problems involving ray construction. Simpler problems involve just the basic phenomena of reflection or refraction, while more complex problems also involve the image concept and location of images. Problems require direct application of basic laws in ray tracing, are of a conceptual and visual nature, and are essentially simulations of physical behavior rather than being formula-based. Relatively little research has been done on student cognition in such optics problems, compared to mechanics for example. The area offers particularly rich possibilities for studying many facets of cognition.

A goal and characteristic of science is to explain the complexity and variety of the natural world in terms of relatively few fundamental concepts and principles. This epistemic nature is emphasized in our inquiry-based physics course for prospective teachers by using an ongoing 'powerful ideas' theme. We hope that students will embrace this perspective not only in learning the fundamentals but also in problem solving, which means that they will consciously apply relevant principles to construct the solution. We will refer to this approach as principle-based reasoning (PBR). It is invariably part of instruction and is reflected in model solutions consisting of systematic stepwise application of principles. While this does indeed represent the physics structure of the final solution, it turns out that it does not reflect the richness of people's cognition when tackling problems. Hsu et. al.[1] recently provided a useful review of work on problem solving in physics; Bodner [2] has noted how chemistry problem solutions likewise do not reflect actual thought processes.

Our study of problem solving in optics suggests that what instructors or experts may regard as involving a straightforward application of physics principles may not be what students see and do as they approach a problem. In our preliminary work it became apparent that many students instead tried to map across remembered results of earlier cases, rather than constructing a principled solution from basics. We use the term case-base reasoning (CBR) to describe an approach where a problem solver retrieves features recalled from similar cases and imports or adapts them to the current problem. Drawing on compiled case knowledge can be very valuable and effective, for both experts and novices, but must be done with awareness of status and applicability conditions [4, 5, 6]. A danger is that novices may simply map across surface features or specific case results, rather than transferring deep structure and procedures.

Note that in physics courses, students encounter both the general principles of a domain and some specific example cases or worked problems. Thus students' overall knowledge from instruction will comprise both general and specific elements. It is only to be expected that both types will play roles in tackling subsequent problems. In addition, students will have their own general thinking tendencies and modes of operating, learned from everyday life and prior schooling, and may also bring experiential intuitions. Instruction of variable quality may or may not have emphasized particular aspects of topic understanding. Given this background, what will happen when a particular student tackles a given physics problem? What reasoning modes and knowledge elements will be cued, in what situations, how, and why? What roles do general principles, procedures, specific cases, result features and intuitions play, and how do they interact? And how does this compare to what experts know and do?

Intrigued by mystifying ray diagrams in student solutions and realizing that they had unexpected difficulties, we undertook a phenomenographic [3] study of reasoning modes in problem solving in optics.

RESEARCH DESIGN

For our course instruction and assessment as well as for the research study we created sets of problems and variants. The problems within each set share a common underlying structure, i.e. solutions are reached by application of the same principles, while situational features differ, leading to differences in result features. For each problem, students provided their written solution along with an explanation of their thinking path and difficulties, and stated how confident they were and why. In addition, after the instructor had discussed solutions of some problems, students wrote reflections about what they had first done and why, and what they had learned from this.

Qualitative and quantitative data sources comprised students' written problem-solving records, individual think-aloud interviews during problem solving, clarification interviews afterwards, and researcher observations during instruction and group work.

ANALYSIS

Guiding questions

Table 1 gives aspects and questions considered in elucidating students' thinking. They emerged as the study developed, and were extended as more problems and data were included. It is useful to list them in two columns reflecting reasoning based predominantly on principles or predominantly on cases, although each usually occurred with a greater or lesser admixture of the other.

Table 1. Analysis Considerations

These questions served to guide our analysis and interpretation of student solutions and thinking.

Regarding principles and their use	**Regarding cases and their use**
• Are concepts & principles/procedures drawn upon? If so, which ones? • Are they appropriately or inappropriately related to the problem? Why? • Declarative: are concepts & principles stated correctly? If incorrectly, how and why? • Procedural: are the principles applied correctly? If not, what errors are made and why? • What role do principles serve in this attempt? To produce the answer? To evaluate a case-based result? To justify a result? • Epistemically aware of role of principles in science?	• Are related cases or previous problems drawn upon? If so, which? • How do they relate to the new problem? Similarities & differences? • What *type* of case knowledge is invoked? Surface features? Result features? Solution method? Other? • What *role* does a previous case result serve? Predict expected result? Provide result directly? A basis for result adjustment? Check on a result obtained from principles? • How is case knowledge used? Mapped 'as is' over to the new? Completely or partially? Mapped but adjusted? • What role does a recalled case *solution method* serve? Procedure to imitate? Guide to implementation of basic principles? • Aware of status and applicability conditions of case features?
Principle-based and case-based aspects: comparison and resolution	
• Are both principle-based and case-based aspects present in an attempt? Do they appear to conflict or agree? • Do students show awareness of comparison or not? Why? • How does apparent agreement or conflict affect students' attempts, as well as their confidence in their work? • If aspects are in conflict, do students try to resolve it? If so, how? Do they try to adjust something to fit? What? • If they cannot resolve a conflict, what do they do? Which mode is 'trusted' more i.e. 'wins', and why?	
Confidence level with reasons	
• How confident is a student about their own solution attempt (be it correct or not) and why?	

Cognitive coding scheme

To analyze and interpret student cognition we developed a coding scheme. The aspects involved and corresponding codes emerged inductively from the study of many students' problem solving attempts enhanced by associated interview data. Problem solutions and thinking were analyzed in 'episodes.' An individual's thinking path through a problem can be represented as a sequence of such coded cognitive elements, and may usefully be displayed diagrammatically in interlinked flow chart form.

RESULTS

The study shows that students bring in a great variety of knowledge elements, at different stages of the problem, for different purposes, and with different intensity. Despite this complexity, it became evident that basic principles and remembered cases both play primary roles in solution attempts. Thus students' thinking may be usefully divided into two main categories: principle-based reasoning (PBR) and case-based reasoning (CBR). Rarely if ever do people use only one of these in its pure form; for novices and experts alike there is usually an interplay of both. An interesting aspect is that students may alternate between PBR and CBR as they work on a problem, or solve part of it one way and part the other, often without being aware of inconsistency.

Example: cognition in a simple reflection problem

When we started studying student thinking on reflection and refraction problems we focused initially on image formation, believing that simple ray tracing problems involving just the basic phenomena would be too easy to bring out many facets of cognition. However, it turns out that even for apparently simple situations thinking can be amazingly varied and interesting, and students have unexpected difficulties. Therefore, to make this point in a short paper, we choose one of the simplest problem variants as an example.

The instructional situation was that students developed the law of reflection in inquiry-based fashion, and practiced applying it for single reflection mirror problems. As a somewhat harder extension, students were given the classic corner mirror problem involving two reflections in succession, to be done by ray tracing. (fig.1.a.). The solution of this case and its practical applications were then discussed in class.

a. **b.**

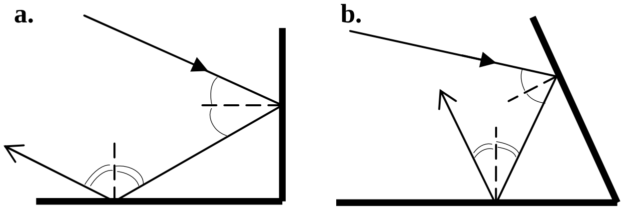

FIGURE 1. **a**. Case encountered in instruction (correct solution)
b. Variant used in examination (correct solution)

In the next test, students were given a problem variant with two mirrors angled as shown in fig.1.b. To trace the ray path, one needs to conceptualize the task as applying the law of reflection twice in succession. This represents principle-based reasoning (PBR).

Student solutions and reasoning

What kinds of ray diagrams do students construct as solutions, and what modes of reasoning lie behind them? We will not describe the whole spectrum of solutions, but refer to the correct diagram above in fig.1.b, and contrast with some incorrect diagrams (fig.2.a. and fig.2.b.) to illustrate interesting aspects of student reasoning.

a. **b.**

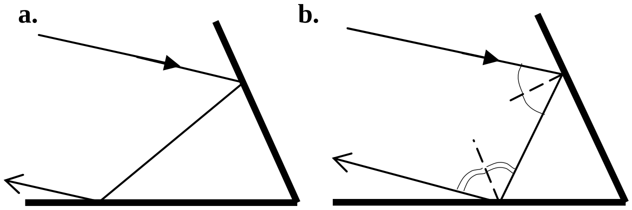

FIGURE 2. **a.** and **b.** Incorrect solutions

As expected for an intrinsically simple problem, a high proportion (78%) of students ended up with a correct solution. However, it is interesting that even where students describe their work in terms of applying principles, interviews and written reflections indicated that they rarely drew on principles alone while solving problems. Cued by similarities with the corner mirror example, students recalled features of the previous case, with various effects, which could be either beneficial or otherwise, depending on whether they also noticed the differences. A variety of scenarios emerged from our data. They are discussed below as primarily principle-based or case-based, or an interplay of the two.

I. PBR as primary approach

1. Principles/procedures recalled via drawing on previous case

"First I thought of how we did the wall reflection problems in class." The similarity and familiarity seems beneficial here, helping a student to implement the desired solution method. Note that she also recognizes the difference between cases: *" ..but this was different because the mirrors were not at right angles."* Thus she can account for the different result features, and has confidence in her principled solution.

2. Solution constructed from principles, with previous case result as expectation or as evaluation

Expectation: *" my first thought was that light will come out parallel in the opposite direction."*

Evaluation: *" I thought about the result [parallel rays] of that [previous] problem as I get to the end."*

The recall of previous case features may confuse some students but may help others. Some students experience an 'aha!' or 'wait a minute!' moment. They compare the two cases, and recognize similarities and differences. They can then accept a principled but unfamiliar result with confidence. *" I checked my work and I told to myself that's [different outcomes] ok because the situations are different."* However conflicts often remain unresolved by the students. In such situations, even with a correct PBR solution (fig.1.b), students lose confidence in their work. *"I thought the lines had to be parallel…in the previous problem similar to this one the lines were parallel…kind of, something like a rhombus… this is what I was picturing… when I did my drawing it ended up being a triangle, they weren't parallel, so I second guessed myself."* She did not consider case difference until it was pointed out *"…I didn't think about that [case differences] until you said it, this mirror is slanted more and in the homework it was just straight up and down, oh!..."*

The above were instances where PBR was dominant and CBR played various secondary roles. Below we consider the reverse situation, where CBR is dominant.

II. CBR as primary approach

1. Result imitation Based on recognized similarities, students interpret a new problem as a familiar case that must have a familiar result. Students start from a previous result as a solution basis, rather than constructing a solution in a principled fashion. In recalling a previous case, students tend to give priority to result features (like the feature of parallel incoming and outgoing rays in our example), rather than relying on common principles. They thus 'map across' results (fig.2.a).

"I remembered seeing something about parallel so I tried to draw them parallel…I sure remembered seeing it in my head, I could visualize [seeing] it on the board."

2. Case result adaptation. When they do recognize case differences (like acute angled mirrors vs. right angled mirrors), students may guess at how to accommodate that difference. For instance, rather than following principled procedures, students simply adjusted the angle of the outgoing ray so that it would no longer look parallel (as it had before).

" this mirror is slanted so it made this one to head a little bit differently, so they are not exactly parallel as they are right here…"

Result adaptation may be preferable to result imitation, but the remaining concern is the absence of epistemic awareness of the need to proceed from basic principles.

III. Interplay of PBR and CBR

A characteristic of the student diagram in fig.2.b. is that it shows a combination of PBR and CBR. The problem naturally splits in two: the first and second reflection. The student uses PBR correctly for the first reflection. However, for the second one, she uses primarily CBR, and reproduces the outgoing ray from the previous case. She also then tries to justify her result by backward application of principles, which forces her to draw an incorrect normal that seems to allow the result she expected.

CONCLUSIONS AND IMPLICATIONS FOR INSTRUCTION

There is a fascinating complexity to thinking and problem solving in science. It is likely that behind the ray diagrams proposed as solutions 'hides' a combination of modes of reasoning based on fundamental concepts and principles, knowledge drawn from cases, and intuitive ideas.

Solving problems by drawing on compiled knowledge from similar cases (CBR) seems to be a strong tendency not only of novices but also of experts [4]. However there is a difference between the way experts use case knowledge and the superficial mode of operating often employed by novices. Properly used, with awareness of applicability conditions, case differences, and generality or specificity, CBR can be a powerful and efficient strategy [5, 6], in both science and everyday life.

Because cognition has so many complex facets the task facing the student is greater than that addressed by prevailing instruction, which focuses mostly on content, not cognition. Certainly some students may not have the required content knowledge to solve a new problem, and re-teaching content may help. However it seems that some students do have a fairly good base of the required declarative and procedural knowledge, but may not call upon it for various reasons, and use a different mode of operating. They quickly interpret the task as a familiar case, even without reading carefully or thinking further, especially in an exam, and hasten to try to reproduce a previous result. This tendency may stem partly from an inappropriate epistemological framework of "what it is all about"; from prior schooling they may have come to view learning as memorization, tests as recall or solving 'seen' problems, and answers as more important than methods.

Implications for more effective instruction are that it should involve not just content but also reflect what we know about real cognition and the nature of expertise. This includes teaching about both principle-based and case-based reasoning, and making such thinking 'visible.' We find it is useful to present two or more problem variations to assist discrimination of both commonalities and differences [7], and to promote explicit reflection, metacognition, and epistemic awareness.

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