

Comparing Explicit and Implicit Teaching of Multiple Representation Use in Physics Problem Solving

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Abstract. There exist both explicit and implicit approaches to teaching students how to solve physics problems involving multiple representations. In the former, students are taught explicit problem-solving approaches, such as lists of steps, and these approaches are emphasized throughout the course. In the latter, good problem-solving strategies are modeled for students by the instructor and homework and exams present problems that require multiple representation use, but students are rarely told explicitly to take a given approach. We report on comparative study of these two approaches; students at Rutgers University receive explicit instruction, while students from the University of Colorado receive implicit instruction. Students in each course solve five common electrostatics problems of varying difficulty. We compare student performances and their use of pictures and free-body diagrams. We also compare the instructional environments, looking at teaching approaches and the frequency of multiple-representation use in lectures and exams. We find that students learning via implicit instruction do slightly better and use multiple representations more often on the shorter problems, but that students learning via explicit instruction are more likely to generate correct free-body diagrams on the hardest problem.

INTRODUCTION

Instructors and researchers in PER have long argued that students can benefit from solving problems that require the use of more than one representation (equations and bar diagrams, for example)[1-5]. These problems are said to require a more complete understanding of the underlying physics than traditional ‘plug and chug’ problems[1,2]. There have been several studies in which students are taught explicit steps for solving problems that use multiple representations [3,4,5]. Another, less-studied approach is to model good problem-solving techniques for students implicitly, without teaching explicit steps. Arguments can be made in favor of either the explicit or implicit approaches. For example, an explicit approach gives students an easy-to-follow ‘checklist,’ though it might also result in dependence on algorithms executed with little understanding. We are unaware of any studies directly comparing explicit and implicit approaches to teaching multiple representation problem solving, and we wish to know whether PER-informed implicit teaching of these skills can be comparable to explicit approaches.

In this study, we consider two introductory algebra-based physics courses taught at large state universities. In one of these (at Rutgers, the State University of New Jersey), the professor taught an explicit set of multiple representation problem-solving steps for physics problems involving forces[6]. These steps were then

emphasized many times over the course of the two-semester sequence. The course at the University of Colorado included many similar multiple representation problems, but did not teach a specific heuristic. Rather, the professor modeled good problem-solving techniques in lecture and in solutions to homeworks and exams.¹

We have two goals in this study. First, we wish to look for differences in performance on multiple-representation problems in the two classes, and for differences in how often and how successfully students in each course used multiple representations in their solutions. Second, we wish to compare the instructional approaches of the two courses as well as the density of representation use in each course, with the intention of relating that to student performance. We can identify three major results so far. First, students from both courses, despite the differences in implementation, use multiple representations in their solutions frequently as compared to students from traditional courses. Second, student performance in each course is very similar. Third, students from the “implicit” course at CU use more representations more often on shorter problems, while students from the “explicit” course at Rutgers are much more likely to draw a complete and correct free-body diagram (FBD) on the most difficult problem.

¹ The differences in multiple representation use are not the only differences between the two courses. Rather, they are the differences we choose to focus on during this study of student use and success with multiple representations.

	Prob. 1	Prob. 2	Prob. 3	Prob. 4
Rutgers	0.38 (235)	0.56	0.32	0.38 (155)
CU	0.38 (314)	0.56	0.43	0.40 (269)

TABLE 1. (Above) Performance of Rutgers and CU students on the recitation questions. The difference on problem 3 is significant ($p=0.008$, two-tailed binomial test). The numbers in parentheses are the sample sizes for problems 1-3 and problem 4.

TABLE 2. (Right) Top: Fraction of students drawing pictures with their problem solutions. Bottom: Average number of forces correctly identified with solutions. Statistical significances are all two-tailed binomial tests.

Pictures	Prob. 1	Prob. 2	Prob. 3	Prob. 4
Rutgers	0.89 (235)	0.89	0.03	0.73 (155)
CU	0.92 (314)	0.91	0.13	0.90 (269)
p-value	X	X	0.0001	<0.0001

Forces	Prob. 1	Prob. 2	Prob. 3	Prob. 4
Rutgers	0.59 (235)	0.52	0.03	1.71 (155)
CU	0.80 (314)	0.71	0.11	1.69 (269)
p-value	0.008	0.05	0.0002	X

We consider these results to be consistent with the course environments, as we argue in the discussion.

METHODS

The study involved second-semester large-lecture algebra-based physics courses from CU and Rutgers, taught in the spring of 2006. Both schools are large public universities, with similar SAT/ACT scores for incoming students. The instructors involved had also taught the first semester of the sequence. Both courses can be described as reformed in nature. A more complete course description can be found in the Data section. Students in each course received a set of four electrostatics problems in recitation that either required calculation of a force or specified forces in the problems. An example problem is included as a note[7]. These problems were ungraded, though students received credit for significant effort.

Students were also given a more challenging problem, intended to be very difficult to solve without an FBD. This problem[8] was given with multiple-choice answers on an exam in the Rutgers course, and was given as a free-response recitation quiz in the CU course.

To complement these problems, we analyzed the representational content of the exams in each course. The procedure (described in [9]) involved quantifying the fraction of lecture time and exam points associated with verbal, mathematical, graphical, and pictorial representations, as well as noting how often multiple representations were used together. We will reserve lecture analysis for a long-format paper, where we will establish that both lectures were rich in representation and multiple representation use.

DATA

We divide our data into four sections. First, we will provide a short description of the problem-solving heuristic taught in the Rutgers course. Second, we shall

consider student performance and representation use on the four recitation problems. Third, we shall examine performance and representation use on the harder exam/quiz problem. Finally, we characterize the representational content of the Rutgers and CU exams. All data include only the students who took both semesters of the physics sequence from the instructor in question.

Course Descriptions. The Rutgers course uses the ISLE curriculum, which is inquiry-based and spends considerable time on the use of multiple representations[10]. In addition, they use the Active Learning Guide workbook in lecture and in recitation, which includes many tasks designed to teach multiple representation use[6]. The instructor teaches students an explicit problem-solving heuristic with five main steps: Picture and Translate, Simplify, Represent Physically, Represent Mathematically, and Evaluate. Students also learn an explicit procedure for constructing free-body diagrams. The CU course includes reforms such as personal response systems (clickers) and PER-based labs and recitation activities[11] (including PhET computer simulations[12]). It also includes substantial multiple representation use in lecture and in homework and exam tasks, but little *explicit* instruction in multiple representation use is given.

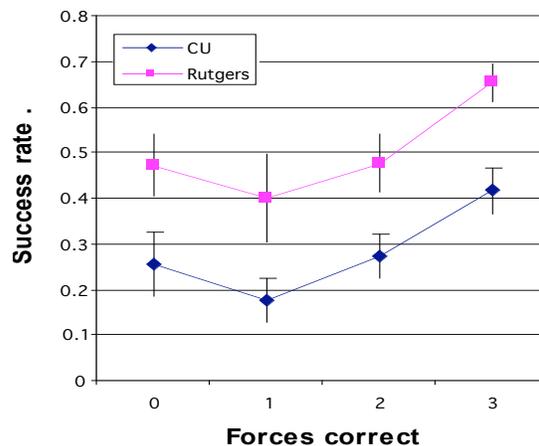
Recitation Problems. Table 1 shows the fraction of Rutgers and CU students answering each of the four recitation questions correctly. The number in parentheses indicates the sample size. Student performance was comparable on problems 1, 2, and 4. On problem 3, 43% of the CU students answered correctly, compared to 32% of the Rutgers students. This difference is statistically significant at a $p=0.008$ level using a two-tailed binomial proportion test. Problem 3 is unique in that it was the only problem that provided a picture and free-body diagram along with the problem statement.

The top of Table 2 shows the fraction of students that drew a picture with their problem solutions. Since problem 3 included a picture, for that problem we show

Exam	Correct	1 Force	2 Forces	3 Forces
Rutgers	0.56 (283)	0.09	0.22	0.51
CU	0.29 (280)	0.23	0.31	0.32

TABLE 3. (above) Fraction of students answering exam/quiz problem correct, and fraction correctly identifying 1, 2, or 3 (out of 3) forces with a free-body diagram. The fractions identifying all three forces correctly are different with $p < 0.0001$.

FIGURE 1. (right) Student performance on exam/quiz problems as a function of number of forces correctly identified. Note that the Rutgers problem was multiple choice, so absolute performances are not directly comparable.



the fraction of students that re-drew their own picture. Picture use is very high on problems 1, 2, and 4, with an overall average of 87% of problem solutions including a picture. CU students drew a picture significantly more often for problem 4.

The bottom of Table 2 shows the average number of correct forces that students identified on their solutions. Since problem 3 included a free-body diagram and only a few students re-drew an FBD, the average number of correct forces is very low, and we instead show the fraction of students re-drawing any forces. We see that students from CU drew more correct forces on average for problems 1 and 2. Note that problem 4 (and only problem 4) asked students to include a complete FBD with their answer, which in that case would include two forces.

Exam/Quiz Problem. Table 3 shows the fraction of students answering the exam/quiz problem correctly at Rutgers and at CU. We also see the fraction of the class that correctly identified one, two, or three forces in their free-body diagrams. An FBD was not required as part of the answer, but the problem was extremely difficult to complete otherwise. As noted previously, the Rutgers problem was given in a multiple-choice format, while the CU problem was given as free-response, so direct performance comparisons are difficult. Since guessing on a five-choice exam should result in a 0.20 success rate, we expect that the CU score would be boosted by that much (or more, since the choices constrain the order of magnitude of the answer) given a multiple-choice format. With that in mind, the CU and Rutgers performances are probably similar.

In Figure 1, we plot problem success versus number of forces correctly identified for both courses. We see the same trend in both courses: Students who identify no forces correctly do as well (possibly better) than those who identify only one. Those that identify two do as well as those who identify none, and those who identify all three do significantly better than all others. The error

bars are large, though overlap is reduced if one averages both data sets together (not shown).

Exam content. In Figure 2, we show the fraction of the exams devoted to different representations and multiple representations. We average over all of the exams from the *first* semester of this physics sequence, as the study took place before any exams in the second semester, and so only those from the first semester could influence these students. We see that the CU exams generally used more representations more often, as compared to the more mathematical exams at Rutgers.

DISCUSSION AND CONCLUSION

We find that the CU group, which had received implicit problem solving instruction, was somewhat more likely to use multiple representations in their solutions to short problems. However, students in the Rutgers group (who received explicit training in multiple-representation problem solving) were much more likely to generate complete and correct free-body diagrams on the most difficult problem (51% vs 32%, as in Table 3). In situations where the performances could reasonably be compared, they were very similar.

Perhaps the most striking result is that students in both courses used multiple representations quite often in their problem solutions (80 or 90 percent of the time). While there were significant differences in implementation, both courses were PER-based and had representation-rich lecture components. For comparison, Van Heuvelen observes much less frequent multiple representation use in traditional courses[3].

Also notable is the observation that CU exams were richer in representations. We have hypothesized previously[9] that representation-rich exams might foster improved representational competence (as such exams hold students accountable for developing broad representational skills), and it is possible that these richer

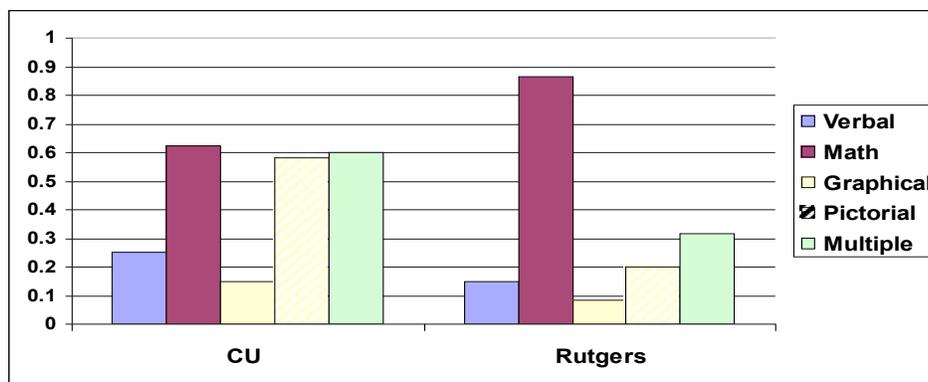


FIGURE 2. Averaged fraction of first-semester exam problems containing listed representations. “Multiple” category refers to the use of two or more representations in one problem.

exams contributed to the CU students’ frequent multiple-representation use on the shorter problems, despite the less explicit problem-solving instruction. More detailed analysis of the course environments is required, and will be reserved for a long-format paper.

We have also seen that the problem-solving algorithm taught to the Rutgers group is detailed and would involve considerably more effort than a standard “plug ‘n chug” approach. This suggests to us an explanation of the differences we observed in representation use. If the students learn to use free-body diagrams and pictures in a specific way, and that way is lengthy, students may be inclined to only use multiple representations when the problem is difficult enough to make the extra effort clearly worth it. Thus, we see the Rutgers students using FBDs much more often on the challenging exam/quiz problem, and using pictures and FBDs less often on the simpler problems. Interestingly, the CU students do not significantly outperform Rutgers students on these shorter problems, raising the question of whether multiple representation use should be valued independent of performance.

It is not clear from these data whether the explicit approach to teaching problem solving was superior to the implicit approach, or vice versa. Indeed, such a judgment may depend on the goals of the instructor. If teaching a specific heuristic were to result different representation use on different tasks, the instructor would need to decide whether that tradeoff was desirable. Without more specific criteria, it appears to us that both appear to have been successful in fostering multiple representation use in solving problems. It is possible that elements of each course could be usefully combined, as well.

ACKNOWLEDGEMENTS

This work was supported in part by an NSF Graduate Fellowship. Special thanks to the PER groups at CU and Rutgers.

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7. A small (100 g) metal ball with $+2.0 \mu\text{C}$ of charge is sitting on a flat frictionless surface. A second identical ball with $-1.0 \mu\text{C}$ of charge is 3.0 cm to the left of the first ball. What are the magnitudes and directions of the forces that we would have to apply to each ball to keep them 3.0 cm apart?
8. A small metal ball with $Q_1 = +2.0 \mu\text{C}$ of charge hangs at the end of a vertical string. A second, identical ball with $Q_2 = -2 \mu\text{C}$ of charge hangs at the end of a vertical string. The tops of the strings are brought near each other, and the strings reach an equilibrium orientation (no longer vertical) when the balls are a distance $d = 3.0$ cm apart. If the force of the Earth on each ball is $F_1 = F_2 = 30$ N, what is the force T of the string on each ball?
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