

# Students' Use of Symmetry with Gauss's Law

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**Abstract.** To study introductory student difficulties with electrostatics, we compared student techniques when finding the electric field for spherically symmetric and non-spherically symmetric charged conductors. We used short interviews to design a free-response and multiple-choice-multiple-response survey that was administered to students in introductory calculus-based courses. We present the survey results and discuss them in light of Singh's results for Gauss's Law, Collins and Ferguson's epistemic forms and games, and Tuminaro's extension of games and frames.

**Keywords:** physics education research, electrostatics, symmetry, Gauss's Law, epistemic frames, epistemic games.

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## INTRODUCTION

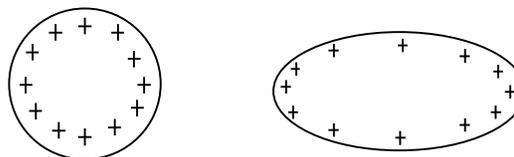
Gauss's Law relates the charge enclosed by a closed surface to the total flux through that surface. In particular symmetry cases (spherical, cylindrical, planar), the flux integral can be simplified and rearranged to find an expression for the electric field. Anecdotal evidence suggests that many introductory physics students use Gauss's Law to find electric fields in a rote way, without concern for the symmetry conditions required to do so. Following initial interviews, we developed a written survey that was distributed in two semesters of the introductory calculus-based electricity and magnetism course. Our intent was to see whether students could recognize the non-applicability of Gauss's Law to a problem situation lacking any of the standard symmetries.

Additionally, our interview results and other research in the field [1] suggest that many students are unnecessarily restrictive when choosing Gaussian surfaces to find the electric flux, while others are not restrictive enough when choosing Gaussian surfaces to find the electric field.

## SURVEY DESIGN

We asked interview and survey subjects about both a spherical charged conductor and a "football-shaped" (prolate spheroid) charged conductor (see Figure 1). Students were asked to sketch the electric field lines and to find an expression for the field both inside and outside the conductor surface, and to explain their reasoning for each step, or, if the field could not be determined, to describe why not. For the written

survey, we also provided students with a list of Gaussian surfaces (sphere, cylinder, cube, football, other) and asked the students to select those surfaces that would be useful for finding the electric field in the given situation.



**FIGURE 1.** Charged, conducting objects for which students were asked to provide field lines and an electric field expression. Students were given that the conductors had total charge  $Q$  and surface areas  $A$  (circle) and  $B$  (football).

Based on our interviews, we expected students to draw reasonably correct field lines for both the sphere and football and either recall expressions for the sphere's electric field or derive them using Gauss's Law. We also suspected that many of the students would use the same electric field method for both the sphere and football, inappropriately generalizing from the spherical case. Although we did not ask about choosing Gaussian surfaces in our interviews, based on Singh's result, we expected some of the students to choose Gaussian surfaces from the list that would not be useful for finding the field.

In the fall of 2005, the survey was offered for nominal extra credit immediately following an exam that covered magnetism ideas. Forty-seven students took the survey, with 45 completing the sphere question and 36 completing the football question. In the spring of 2006 the survey was administered in two

out of 8 recitation sections; the TAs for these sections offered to administer the survey during class time. The survey was distributed following an exam that covered Gauss's Law, but no extra credit was offered. Sixteen students completed the survey. All students completed both questions. (The groups will be referred to by year, i.e., 2005 and 2006.)

## RESULTS AND DISCUSSION

Although we were primarily interested in how students developed a mathematical expression for the electric field, many students had difficulty with the supporting ideas required to begin such a development, including the differences between electric and magnetic ideas and the construction of field lines.

A few students in 2005 seemed to use magnetic ideas in tackling the problem. One student drew X's around the conductor and drew perpendicular force and velocity vectors, although she referred to these as "electric field lines." We interpret the X's as indicating a magnetic field directed into the page. Other students wrote expressions such as  $\oint \vec{E} \cdot d\vec{l} = \frac{\epsilon_0 I_{\text{induced}}}{2\pi}$ , apparently confusing Gauss's Law with Ampère's Law. These students, however, were in the minority.

A number of students showed trouble with one or more ideas from electrostatics. For example, even in the simpler case of the sphere about 40% of students in 2005 ( $n = 45$ ) drew field lines with some kind of error (curved rather than straight, inside the conductor, or pointing toward the conductor). A third of the students gave a nonzero field expression for the inside of the conductor and 18% gave an expression for the field outside the conductor with no dependence on distance. Surprisingly, 18% also drew or described negative charges attracted by the positive charge on the conductor. These negative charges sometimes resided on the Gaussian surface, indicating an incomplete understanding of the purpose of the Gaussian surface. All of these difficulties—confusion with magnetic ideas, trouble drawing field lines, and qualitatively incorrect field behaviors inside and outside conductors—would make appropriate use of Gauss's Law difficult.

In 2005, only 7 of 45 students wrote a correct expression for the electric field outside the spherical conductor, and of these students 4 included some form of correct explanation. In 2006, 10 (of 16) gave the correct expression; of these, 3 accompanied it with a correct explanation.

The football question was even more difficult for students. In 2005, 5 of 36 students gave what we considered a correct answer for the field outside the football (that is, the electric field cannot be found), but

none provided a correct explanation of why this was the case. One student, however, seemed to be on the right track with the explanation "the electric field is not evenly dispersed so without the dimensions the electric field cannot be found." Students in 2006 fared no better, with 3 of 16 stating that the electric field could not be found. Two of these students explained their response in a similar manner to the student in 2005.

**TABLE 1.** Percentage of correct electric field expressions for the two conductors (with or without correct explanation).

Field Location	2005 semester		2006 semester	
	Sphere (n=45)	Football (n=36)	Sphere (n=16)	Football (n=16)
Inside	29%	36%	44%	56%
Outside	16%	14%	63%	19%

Overall, fewer than half of the students explained the reasoning behind their responses on the written survey in 2005, either with words or with a step-by-step mathematical solution, for either correct or incorrect responses. In 2006, we strengthened language in the directions explicitly asking for a written explanation, but still, only 9 of the 16 students provided any sort of explanation. Even from the scant information supplied, however, some interesting patterns emerge.

The rarity of step-by-step problem solving was a more widespread issue. Very few students provided any mathematical steps leading to their final electric field expressions. This suggests that in the case of the sphere they simply remembered and wrote the  $kq/r^2$  form for a spherical charge distribution.

Although the data were somewhat "noisier" than we expected, the predicted pattern did emerge. In particular, of the students who gave electric field expressions for both problems (34 students in 2005 and 16 in 2006), 66% in 2005 and 57% in 2006 used essentially the same method for both (i.e., the same steps if a derivation was used, or the same functional form if only an equation was given).

The results of the multiple-choice-multiple-response Gaussian surface question, summarized in Table 2, were as expected. For the sphere, 40% (2005) and 75% (2006) of students chose only a spherical Gaussian surface, but a significant minority (31% and 13%, respectively) included some, but not all, other listed surfaces in addition to the sphere. Most frequently, the additional surface was the cylinder. Additionally, 24% and 12% of students chose either all the listed surfaces or explicitly wrote that any closed surface would be a useful Gaussian surface.

For the football, 25% (2005) and 75% (2006) of students chose only a football-shaped Gaussian surface. Fewer students (8% and 13%) chose the football along with some other surfaces, and 25% and 12% of students chose all the listed surfaces or any

closed surface. Interestingly, although only 2% of students in 2005 (and none in 2006!) did not include the spherical Gaussian surface when choosing appropriate surfaces for the sphere, 17% and 6% of students did not include the football-shaped surface in the football situation.

**TABLE 2.** Shapes of Gaussian surfaces chosen for both shapes of conductor

Surface Chosen	2005 semester		2006 semester	
	Sphere (n=45)	Football (n=36)	Sphere (n=16)	Football (n=16)
Shape Only	40%	25%	75%	75%
Shape + others	31%	8%	13%	13%
Other than shape	2%	17%	0%	6%
Any or all	24%	25%	12%	12%

The results of the Gaussian surface question indicate several difficulties with the interpretation of Gaussian surfaces as a (mathematical) tool.

### Interpretation of Results Using Epistemic Frames and Games

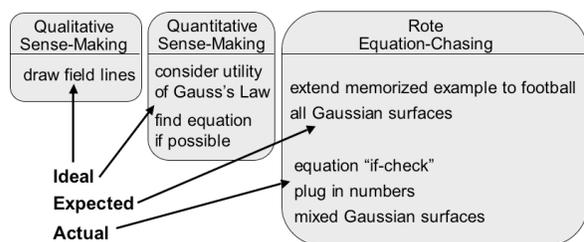
Epistemic frames [2,3] and games [3,4] as discussed in earlier work and then applied to physics problem solving [3], provide an interesting lens through which to view our data. Briefly, an epistemic frame can be described as a student's perception and expectations surrounding the situation at hand (for example, being in a lecture setting, or doing physics homework in a dorm room). An epistemic game is the series of moves followed by the student (e.g., writing notes and staying quiet in a lecture, or working through the solution to the physics problem and asking friends for help) to generate some target structure, known as the epistemic form [4]. (In a lecture setting, the epistemic form associated with the *note-taking* game is a set of notes that reflect the lecture content.) A student's current frame influences which games are seen as useful in order to achieve the perceived target structure.

We interpreted our results using this framework, in the hopes of gaining additional insights into student responses. Tuminaro identifies three distinct frames students use when solving physics problems: *rote equation-chasing*, *qualitative sense-making*, and *quantitative sense-making*.

To successfully navigate both survey tasks, students must use the *qualitative* (for the field line sketch) and *quantitative* (for the field expression) *sense-making* frames. For the sphere, *rote equation-chasing* could potentially lead to a correct response. Since using any memorized general form of Gauss's Law (including restricted forms such as  $EA = q/\epsilon_0$ ) leads to the correct solution, students do not need to make sense of Gauss's Law or consider if it applies in order use it correctly. For the football-shaped

conductor, however, the electric field is stronger near the "ends" of the football. Because of this lack of symmetry, it is essentially impossible to find a Gaussian surface for which the magnitude of  $E$  is constant without already having an expression for the field, rendering Gauss's Law impotent in this situation. Students in the *rote equation-chasing* frame might simply extend their solution from the sphere task, while students in a *sense-making* frame might realize that the change in the problem situation prohibited a straightforward use of Gauss's Law to find the field.

In general, we expected many students to be in the *rote equation-chasing* frame, remembering a series of steps to use Gauss's Law for the sphere and applying those same steps to the football. We found that many students were even more rote in their use of the process than we expected, to the point of some refusing to answer without numbers to substitute for the values of the charge on the conductors. Many students seemed to process a kind of "if-check": if the electric field equation was remembered, then write it down; otherwise, don't worry about deriving it. Figure 2 summarizes a frames view of the survey tasks and results.



**FIGURE 2.** Epistemic frames and associated epistemic games associated with the task of determining the electric field for the objects in Figure 1.

The idea of epistemic forms also provides one possible explanation for the lack of student explanations when completing the survey. If students perceived the expression for  $E$  to be the target structure, then detailed steps would not necessarily be worth writing (especially if the final equation could simply be recalled). Since the grading of homework and exam questions in our traditional courses typically emphasize the final result (even if the problem asks for explanation), it is not unreasonable to suspect that students considered providing reasoning as unimportant, rather than being unable to provide reasoning. For the researchers, on the other hand, the process was as interesting as the final expression, so the complete derivation was the hoped-for epistemic form. If further surveys were done in the future on this topic, it might be helpful to rewrite the language to de-emphasize the importance of the final field expression.

## Complicating Factors

Our choice of survey instruments, when examined in the light of epistemic frames, contains a dilemma: is the use of both “working” and “non-working” cases of Gauss’s Law an unfair trick to play on students? If they recall how Gauss’s Law is used in the case of the spherical conductor, that first task could cause students to overgeneralize this method to the football conductor without careful thought. In addition, in traditional physics classes (such as those from which we drew our subjects) the correct answer is rarely “you can’t do this problem,” further encouraging the application of the solution for the sphere to the football.

On the other hand, if students are prompted into using Gauss’s Law inappropriately, it indicates that they may use the law in an algorithmic way rather than as a sometimes-useful tool. The prevalence of this behavior was exactly what interested us; thus potentially priming students with the sphere was an acceptable possibility.

One other pattern of interest that emerged from the data was the generally better performance of the students in 2006. For example, students in 2006 did not reason using magnetic ideas or write  $E$ -field expressions that did not depend on distance from the conductor, and they used the idea of induced charge far less frequently. One possible explanation is that there were far fewer students participating in 2006 as in 2005; the smaller number of respondents might have magnified quirks in the data. Alternately, significant ability differences between the two classes might exist. In our view, however, the most likely cause of the improvement in 2006 was timing. The 2005 students were given the survey late in the course, after an exam covering magnetism, while the 2006 students received the survey after an exam that included Gauss’s Law problems. Not only was the material fresher in the minds of students in 2006, they had no instruction on magnetism to confuse with electrostatic topics.

## CONCLUSIONS

Student performance in deciding how to determine the electric field of spherical and prolate spheroidal charged conductors suggests that Gauss’s Law and the underlying procedures and concepts, especially the restrictive applicability of Gauss’s Law, are not well understood by many introductory students.

Although the sample size was small, we noticed a qualitative difference in the types of responses between semesters. Some responses in one group seemed influenced by course content introduced between Gauss’s Law and the administration of the survey. This suggests that while students may understand some of the consequences of Gauss’s Law

at the time that it is taught, newer electric and magnetic concepts can be confused with or replace the earlier content. While confusion between electric and magnetic ideas has been documented previously [5], it has not been seen with Gauss’s Law.

The overall pattern of matched responses was in line with our expectations in type, if not in degree: most students who attempted both problems used the same method for both, indicating that they do not consider the shape of and charge distribution on a conductor to be an important cue. However, very few students made an attempt to derive an expression for the electric field if they could not recall it, as we had hoped they might. One possible explanation for this lack of written reasoning is a mismatch of epistemic frames between students and researchers, with few of the students considering documentation of the problem-solving process as an important part of the solution.

Finally, we partially replicated Singh’s result: many students are too permissive when choosing appropriate Gaussian surfaces to find electric fields.

Gauss’s Law can be a powerful tool, but it requires a deep understanding of both physical and mathematical principles in order to use it effectively. Students who lack this understanding are unlikely to use (or not use) Gauss’s Law appropriately in unfamiliar situations.

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