

Probing Student Reasoning and Intuitions in Intermediate Mechanics: An Example With Linear Oscillations

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Abstract. The study of linear oscillations—including simple harmonic, damped, and driven oscillations—is not only fundamental in classical mechanics but lies at the heart of numerous applications in the engineering sciences. Results from research conducted in the context of junior-level mechanics courses suggest the presence of specific conceptual and reasoning difficulties, many of which seem to be based on fundamental concepts. Evidence from pretests (ungraded quizzes) will be presented to illustrate critical difficulties in understanding conceptual underpinnings, relating concepts to graphical representations (*e.g.*, motion graphs), and connecting the physics to the relevant differential equations of motion. Preliminary results from the development of a tutorial approach to instruction, modeled after *Tutorials in Introductory Physics* by McDermott, *et al.*, [1] suggest that such an approach can be effective in both physics and engineering courses. (Supported by NSF grants DUE-0441426 and DUE-0442388.).

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INTRODUCTION

As part of an ongoing investigation of student learning in intermediate mechanics, we are probing how advanced undergraduate majors in physics, math, and engineering think about oscillations in one and two dimensions. Instructors often expect their students to extend what they have learned at the introductory level about oscillatory motion (*e.g.*, simple harmonic motion) to situations that are physically and mathematically more sophisticated. However, evidence from this study corroborates previous studies that demonstrate how difficulties with basic concepts can hinder meaningful learning in upper level courses [2]. Furthermore, analysis of student responses to numerous research tasks, including written qualitative questions that require explanations of reasoning, often suggests that students need guidance in organizing their knowledge.

This report will focus specifically on the following research questions: (a) How well do students understand the factors that affect the *frequency* of different types of linear oscillations? (b) How well do students interpret and understand *formal representations* of oscillatory motion, such as x vs. t graphs of 1-D oscillators and x - y trajectories of 2-D oscillators? (c) To what extent do students answer qualitative questions by bringing to bear their

knowledge of *general principles* relevant to the physical situation at hand?

CONTEXT OF INVESTIGATION

The student populations discussed here come from junior-level intermediate mechanics courses at Grand Valley State University (GVSU), the University of Maine (UME), and Seattle Pacific University (SPU). Although the details of the courses vary somewhat, all courses cover linear oscillations (simple harmonic, damped, driven) and other topics that require the synthesis of Newton's laws, work and energy, and differential equations. In addition, the classes discussed here were taught either by the author or Michael Wittmann (UME), with whom the author is collaborating on *Intermediate Mechanics Tutorials (IMT)*, a set of research-based curricular materials modeled after *Tutorials in Introductory Physics* [1].

The results presented in this paper were taken primarily from the analysis of responses to written pretests (ungraded quizzes). In all cases the pretests were given after lecture instruction but before the tutorial (from *IMT*) on the relevant topic. At GVSU and SPU each pretest was administered during class for 10 min; at UME students were instructed to complete each pretest outside class for 10-15 min. All pretest questions asked for explanations of reasoning.

PROBING STUDENT THINKING OF SIMPLE HARMONIC MOTION IN ONE AND TWO DIMENSIONS

In this section we describe results from pretests that probe the ability of students to apply (in 1D) and extend (to 2-D) the idea that the frequency [$\omega_0 = (k/m)^{1/2}$] of a simple harmonic oscillator is determined solely by the spring constant and mass. Students in all classes discussed here covered 1D simple harmonic oscillators at the introductory level.

Simple Harmonic Motion

The first pretest on oscillations includes questions that elicit student ideas about the factors that affect the frequency of simple harmonic oscillations. For this report we describe the results from 4 classes ($N = 35$) at GVSU and 1 class ($N = 11$) at SPU.

Students are shown a strobe picture illustrating a block connected to an ideal spring that is released from rest on a level, horizontal surface. They are asked how the period would be affected by: (i) changing the release point of the block from 0.5 m to 0.7 m from equilibrium, (ii) replacing the original spring with one that is stiffer, and (iii) replacing the original block with one having four times the mass. The students were expected to recognize that the period will not change in case (i), decrease in case (ii), and increase (double) in case (iii).

Incorrect Intuitions Relating Period and Amplitude

Although most students gave correct responses (ignoring reasoning) for each case, case (i) yielded the lowest percentage of complete and correct explanations (~10%). Many correct responses were supported by “compensation arguments” [3] relating amplitude, average speed, and period. As one student explained, “It may seem that the block is moving faster, but it is also moving farther to compensate.” While such justifications make it *plausible* that the period is unaffected by changing the amplitude, they show no evidence of understanding that *only* the spring constant and mass affect the period. Even more telling, the most common *incorrect* explanation (from ~25% of the students) was based on the incorrect intuition that the greater initial displacement from equilibrium (and hence the larger amplitude) would cause the period to increase because, for example, “the block travels farther during each period.”

The above results are interesting because they suggest persistent, incorrect intuitions that may lead to confusion in the context of 2-D oscillations. Even though students completed a tutorial (not discussed

here) on 1-D harmonic oscillators, the above pretest results suggested the need to explore how students proceed from 1-D to the 2-D case.

Harmonic Motion in Two Dimensions

Students often were introduced to 2-D oscillations as an application of conservative forces, several weeks after covering 1-D oscillations. The following pretest, given after relevant lectures to 4 classes ($N = 31$) at GVSU and 1 class ($N = 17$) at UME, was designed to probe student understanding of the relative frequencies along the x - and y -axes of a 2-D oscillator.

Students are asked to consider an undamped 2-D oscillator with $U(x, y) = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2y^2$. (They are also reminded about the relationship $k_i = m\omega_i^2$ for each force constant.) For each x - y trajectory shown in Fig. 1, students are asked whether that trajectory is possible for such an oscillator and, if so, whether ω_1 is greater than, less than, or equal to ω_2 . (The original version of the pretest asked for a comparison of the force constants k_1 and k_2 instead of the frequencies. The change in wording, however, did not significantly alter overall student performance.)

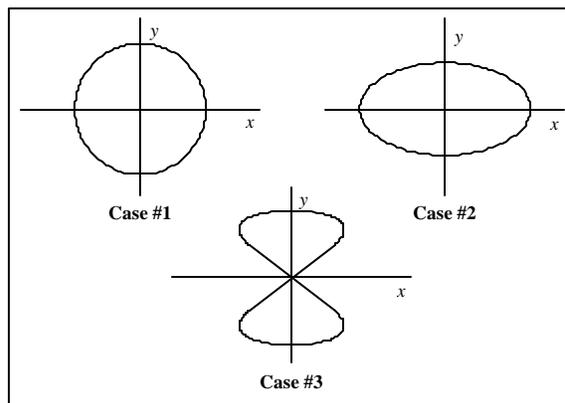


FIGURE 1. Three x - y trajectories shown on the pretest for 2-D oscillators. Students were asked for each case how the frequencies along the x - and y -axes compared.

Students were expected to infer from each trajectory how many cycles occurred along one axis for each cycle along the other. Two isotropic cases (#1 and #2) were included, and showing different x - and y - amplitudes for Case #2 was intended to elicit incorrect intuitions about frequency and amplitude.

Use of “Compensation Arguments” Relating Frequencies (or Force Constants) and Amplitudes

In each class very few students (between 0% and 15%) gave correct responses for all cases, even when

explanations were ignored. Most students incorrectly compared the relative frequencies (or relative force constants) by using inappropriate “compensation arguments” involving the relative amplitudes along the x - and y -axes. For example, for case #2 most incorrectly predicted that $w_1 < w_2$ (or $k_1 < k_2$) for reasons such as: “the spring goes farther in the x -direction, so [the] spring must be less stiff in that direction,” or “since we now have an oval curve with the x -axis longer, w_2 must be greater to compensate.”

The prevalence of this type of “compensation” reasoning is striking for two reasons. First, it strongly suggests that most students fail to recognize that x - y trajectories like those from the pretest yield frequency information about the 2-D oscillator. Second, the tendency for students to link amplitudes with frequencies (or force constants) appears to be analogous to the most common incorrect mode of reasoning used on the 1-D oscillator pretest. This result suggests the recurrence of conceptual difficulty with fundamental ideas.

PROBING STUDENT THINKING OF DAMPED HARMONIC MOTION

Students encountering damped oscillations for the first time usually do so at the intermediate level, after simple harmonic motion but before 2D oscillations. Typically the lecturer demonstrates shows how to set up and solve the differential equation. For the underdamped form of the solution the students are shown that amplitude decreases exponentially with time and that the frequency is smaller than that for the undamped oscillator: $w_d = (w_o^2 - g^2)^{1/2}$, where g is the damping constant. Given this typical treatment of damped oscillators it was desired to study how well students understood qualitatively how the presence of damping affects the motion of an oscillator.

Underdamped Motion

After lecture instruction on damped oscillators, students in 4 GVSU classes ($N = 35$) and 1 SPU class ($N = 11$) were given the following two-part pretest. The pretest began by showing students the x vs. t graph of a simple harmonic oscillator (no damping) released from rest (see solid curve in Fig. 2). They were then told to assume that a linear damping force is applied, causing the oscillator to become underdamped. (Students were reminded of the meaning of the term.) In part A of the pretest, students were asked to sketch a qualitatively correct graph of the underdamped oscillator having the same initial conditions as the original (undamped) one. In part B, they were asked to consider the instant it first passes $x=0$: at that

instant is the oscillator speeding up, slowing down, or moving with maximum speed?

(Note: Part B was not included on the pretest for one GVSU class. A slightly different version of part B was given to two of the GVSU classes: Does the [underdamped] oscillator first attain a maximum speed before, after, or exactly at the same instant when it passes through $x=0$? The change in wording did not seem to affect the overall performance of the students.)

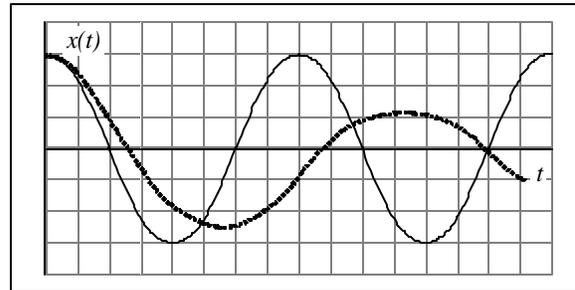


FIGURE 2. Motion graph from part A of the pretest on underdamped oscillators. The solid curve represents the motion of a simple harmonic oscillator. The dashed curve (not shown to students) illustrates a qualitatively correct graph for an underdamped oscillator.

Inappropriate Generalizations from the Case of Simple Harmonic Motion

Few students (~25% or fewer) in each class answered part A correctly. Any curve like the dashed curve shown in Fig. 2 would have been acceptable. However, most students (60% to 70%) drew graphs like the one shown in Fig. 3, showing a gradually decreasing amplitude (which is correct) but a frequency that is equal to that for the undamped case. Most explanations—for example, “the amplitude shrinks in time but the period shouldn’t change since they are independent of each other”—suggest an overgeneralization from simple harmonic motion.

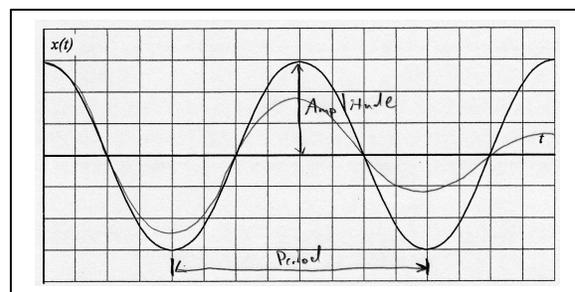


FIGURE 3. Example of a typical incorrect student graph elicited by the pretest on underdamped oscillations. Most students drew graphs like this one, showing equal frequencies for the undamped and underdamped cases.

Other errors arose on part A, including the tendency to show *both* the amplitude *and* period as gradually decreasing. These responses could be interpreted as recurrences of the belief that amplitude and frequency are connected, a belief that was detected on each of the two pretests described previously. More research is needed, though, to tell for certain.

Part B of the pretest was equally difficult for students. Students could answer part B by taking the differential equation of motion, $ma = -cv - kx$, setting $x = 0$, recognizing that acceleration and velocity must be opposite in direction, and concluding that the oscillator must be slowing down at $x = 0$. Students could get the same answer by drawing a free-body diagram and finding that the net force opposes the velocity.

Only 20% to 30% of the students in each class gave correct responses. The most common incorrect answer was to state that the oscillator experienced its maximum speed upon passing $x = 0$. Some did not seem to take the damping into account, saying that there was no acceleration because the spring was neither pushing nor pulling. Others did not at all invoke forces or Newton's laws, saying simply that the slope of the x vs. t graph would be at a maximum at $x = 0$. Both modes of reasoning strongly suggest a tendency to overgeneralize from the case of simple harmonic motion rather than to bring to bear one's knowledge of Newton's laws.

IMPLICATIONS FOR INSTRUCTION AND FUTURE RESEARCH

Although the pretests discussed here do not necessarily measure depth of student understanding, they show what physics majors often cannot do after traditional lectures. They need guidance in recognizing which factors affect the frequency of various types of oscillations, including simple harmonic motion (covered at the introductory level) and underdamped motion (covered usually for the first time at the sophomore or junior level). Students also have difficulty interpreting representations of oscillatory motion, including x - y trajectories of 2-D oscillators. As with many investigations conducted at the introductory level and beyond, traditional lecture instruction has found to do little to promote conceptual development in students, even students who are physics majors [4].

The prevalence of incorrect or incomplete explanations suggests that many students entering the intermediate mechanics class lack a strong conceptual framework upon which to build. Rather than recognize the relevance and utility of physics principles (*e.g.*, Newton's second law), many students

tend to make inappropriate generalizations from special cases (*e.g.*, to incorrectly infer the behavior of underdamped oscillators from results that are valid only when damping is absent).

Although some types of incorrect responses, including "compensation arguments" linking amplitude to frequency (or period) are prevalent, it is possible that they may not indicate hard-and-fast conceptual difficulties as much as the tendency for students to proceed from incorrect assumptions or even inadvertent "triggers" from the research task. For instance, modifications are being considered for the 2-D oscillator pretest in which the students will not be shown a set of possible x - y trajectories. In the event that presenting both circular and elliptical trajectories "triggered" a high percentage of amplitude-frequency explanations, students will instead be given the initial conditions of motion and a specified k_2/k_1 (or w_2/w_1) ratio. They will then be asked to sketch a possible x - y trajectory for the oscillator. It is hoped that analysis of student responses on the revised pretest will allow a measure of the robustness of amplitude-frequency explanations. Such results would be used to guide refinements to existing *IMT* materials, so that they will become even more effective in helping students make the qualitative and quantitative extensions from introductory to intermediate level mechanics.

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REFERENCES

1. L. C. McDermott, P. S. Shaffer, and the Phys. Ed. Group at the U. of Washington, *Tutorials in Introductory Physics*, Prentice-Hall, Upper Saddle River, NJ, 2002.
2. B. S. Ambrose, *Am. J. Phys.* **72**, 453-459 (2004).
3. T. O'Brien Pride, S. Vokos, and L. C. McDermott, *Am. J. Phys.* **66**, 147-156 (1998).
4. L. C. McDermott and E. F. Redish, *Am. J. Phys.* **67**, 755-767 (1999).