Appendix C: Supplemental Calculations for Model Evaluations and A New Multiple-Choice Instrument on Newton III with Isolated Physical Features

Relative Model Mixing Rate

In chapter 4, $\gamma_{\eta\mu}$ is defined as

$$\gamma_{\eta\mu} = \frac{\rho_{\eta\mu}}{\sqrt{\rho_\eta \rho_\mu}} \qquad \text{(see 4-23)}$$

Without losing generality, let's use a three-model example to show that $\gamma_{\eta\mu}$ gives a value between 0 and 1. Suppose we can a total of N students and the measured class density matrix is

$$\mathcal{D} = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix}$$

Representing the measured k^{th} student model state with

$$\mathbf{u}_k = \begin{pmatrix} a_k \\ b_k \\ c_k \end{pmatrix}$$

Consider the mixing between the first and second physical models. For the actual measured class model density matrix, we have:

$$\rho_{11} = \sum_{k=1}^{N} a_k^2 , \quad \rho_{22} = \sum_{k=1}^{N} b_k^2 , \quad \rho_{12} = \sum_{k=1}^{N} a_k b_k$$

In order for $\gamma_{\eta\mu}$ to have a value between 0 and 1, we need to satisfy the condition

$$\sqrt{\rho_{\eta\eta}\rho_{\mu\mu}} \geq \rho_{\eta\mu}$$

which is translated to

$$\left(\sum_{k=1}^{N} a_k^2 \cdot \sum_{k=1}^{N} b_k^2 \right)^{1/2} \geq \sum_{k=1}^{N} a_k b_k$$

$$\therefore \quad (a_1^2 + a_2^2 + \ldots + a_N^2) \cdot (b_1^2 + b_2^2 + \ldots + b_N^2) \geq (a_1 b_1 + a_2 b_2 + \ldots + a_N b_N)^2$$

$$\therefore \quad \sum_{j=i+1}^{N}\sum_{i=1}^{N}(a_i b_j - a_j b_i)^2 \geq 0$$

which is always true. The equal sign holds when $a_i b_j = a_j b_i$, which requires that $a_i = a_j$ and $b_i = b_j$. This means that all students have the same model state.

Additional Numerical Evaluations of Student Model States

1. Model Energy Distribution and Net Model Energy

We know that typically the new eigenstates of the density matrix have components on all the bases. The agreement gives a good assessment on how well the student models agree with the base models. It will also be useful to know how the elements of each class model state are distributed. This is in analogy to an energy distribution where we define $v^2_{\eta\mu}$ as the model energy. For each class model state, we can construct a model energy plot that illustrates the distributions. Taking $\mathbf{v}_1$ in Eq. (4-19) as an example, the model energy for that state is shown in figure C-1.

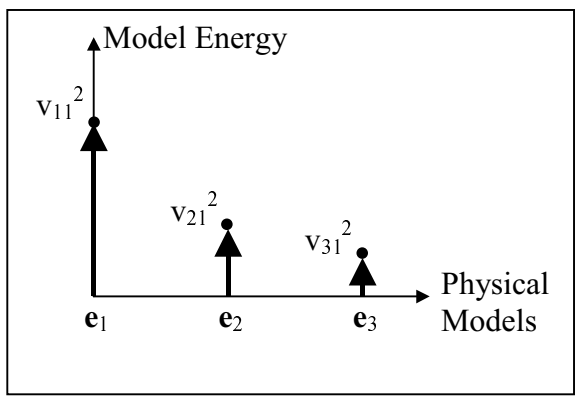


Figure C-1. Model energy distribution

Define the *net model energy* as the multiplication of the model energy and the eigenvalue of the model vector. The net model energy gives the net contribution on physical models from a student model states. Thus for the example shown in figure C-1. the net model energy components for $\mathbf{v}_1$ are $\sigma_1^2 v_{11}^2$, $\sigma_1^2 v_{21}^2$ and $\sigma_1^2 v_{31}^2$ respectively.

2. Model Mixing Quality

As shown in figure 4-9, we call the peak with the largest energy as the main lobe and the ones with smaller energy as side lobes. The main lobe energy represents the effects from the closest base model and the side lobe energies indicate the mixing from the other base models. The main lobe energy has the same value of the agreement measurement and the side lobe energies can be considered as the cross model overlap. Since for the same agreement value, there can be different values for side lobe energy, the ratio of the main lobe energy to the largest side lobe energy can be an evaluation for the model mixing

quality. Large ratio indicates less mixing. Define the ratio as γ_E. For the example in figure 4, the γ_E can be calculated with

$$\gamma_E = \frac{v_{11}^2}{\mathrm{Max}(v_{21}^2, v_{31}^2)}$$

where Max() returns the maximum value from a number of inputs.

3. Modeling Condition

As we have discussed in the previous sections, the distribution of the eigenvalues provides the information on the consistency of the behaviors of different students. Therefore a large dominant eigenvalue is an indication of a large body of students with very similar behavior and thus a good modeling condition for the class. We can design a numerical measure, γ_M, to evaluate this modeling condition. It is defined as the ratio between the largest eigenvalue and the second largest eigenvalue. For the example of a 3-D model space, the modeling condition can be calculated with

$$\gamma_M = \frac{\sigma_1^2}{\mathrm{Max}(\sigma_2^2, \sigma_3^2)},$$

where σ_1^2 represents the largest eigenvalue.

Model plot of the individual classes on N3 and FM group

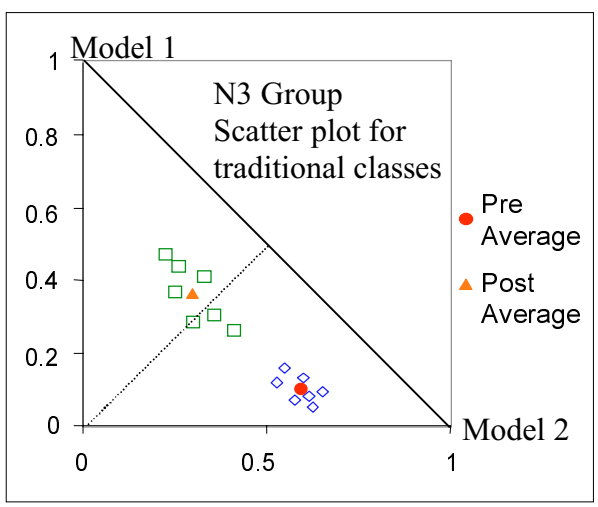


Figure C-2. The primary class model states on Newton III (FCI test) of 7 UMd classes with traditional instruction

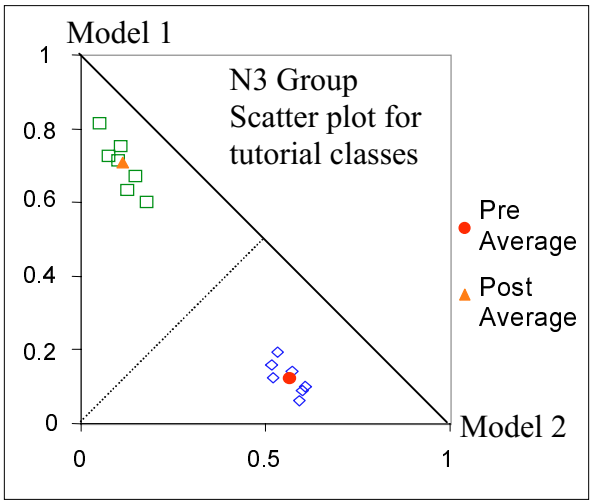


Figure C-3. The primary class model states on Newton III (FCI test) of 7 UMd classes with tutorial instruction

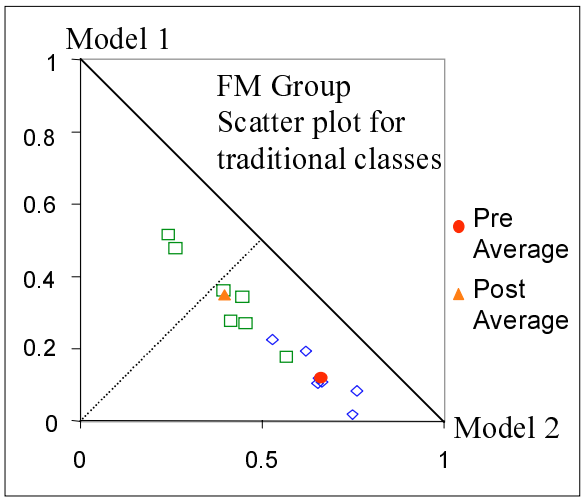


Figure C-4. The primary class model states on Force-Motion (FCI test) of 7 UMd classes with traditional instruction

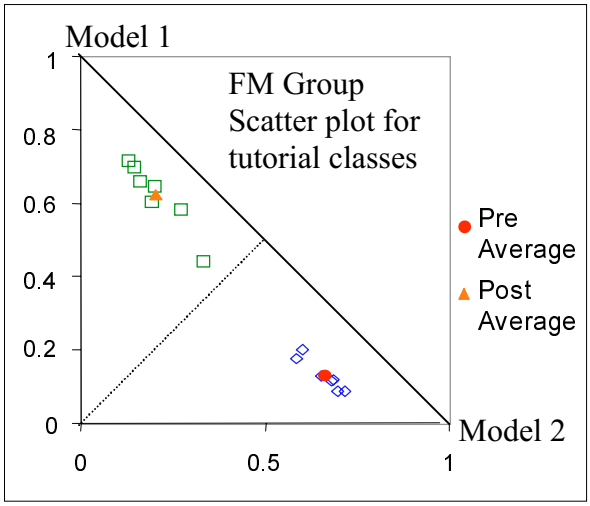


Figure C-5. The primary class model states on Force-Motion (FCI test) of 7 UMd classes with tutorial instruction

Survey on Newton's Third Law

Instructions:

- *Please read each question carefully. If you have any questions, ask the instructor for help.*
- *For each question, please only select **one** choice that best describes your understanding.*
- *If none of the choice matches what you think (choice "F"), please **do not** mark any choices on the computer card and write choice "F" next to the question. Please also write your own comments in the space between questions or on the back of the questionnaire sheet.*
- *This test is for diagnostic purpose only and will not influence your grade in any way. Therefore, we would like you to give your true thoughts on the physics involved. This will greatly help us to design better instruction and help you to improve your performance on exams.*
- *Your name and ID on this survey are for administration purposes, and your result will be kept strictly confidential.*

Use the following choices to answer questions **1 – 4**.

A. The truck exerts a greater force on the car than the car exerts on the truck.

B. The car exerts a greater force on the truck than the truck exerts on the car.

C. The truck exerts a force on the car but the car doesn't exert a force on the truck..

D. The car exerts a force on the truck but the truck doesn't exert a force on the truck.

E. The truck exerts the same amount of force on the car as the car exerts on the truck.

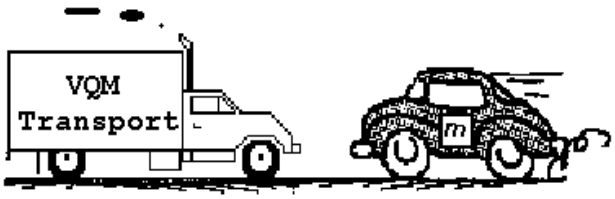
F. None of the answers above describes the situation correctly.

1. As shown in the figure on the right, a collision happens between a small pickup truck and a car. The small truck has the **same weight** as the car does. At the time of collision, both vehicles travel at a constant speed but the small truck is moving at a **slower speed** than the car. When they collide, which choice describes the forces?

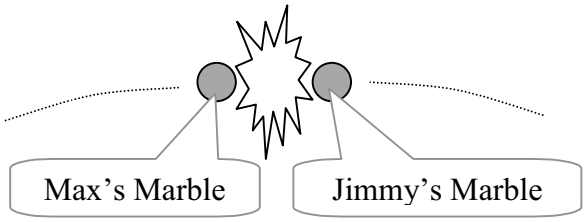
2. Now consider the case that before the collision, the car is traveling at a constant speed while the truck starts slow and is **speeding** up. At the moment of collision, both vehicles happen to be traveling at **the same speed**. When they collide, which of the above choices describes the forces?

3. As shown in the figure below, a car breaks down on the road and is pushed into town by a small AAA service truck. The small truck has the **same weight** as the car does. While the truck, still pushing the car, is **speeding** up, which choice describes the forces?

4. As shown in the figure below, a collision happens between a big truck and a car. The big truck has **a much larger weight** than the car does. Before the collision, both vehicles are traveling at a **same constant speed**. When they collide, which choice describes the forces?



5. Two boys, Jimmy and Max, are tossing **identical** marbles at each other. The Max is a little stronger than the Jimmy, so the marble from him goes **faster** than the marble from the Jimmy. During one nice shot, the two marbles, one from the Max and one from the Jimmy, collide in mid air (see figure below). Which of the following statement is true about the force?



A. Max's marble exerts a greater force on Jimmy's marble than Jimmy's marble exerts on Max's marble.

B. Max's marble exerts a same amount of force on Jimmy's marble as Jimmy's marble exerts on Max's marble.

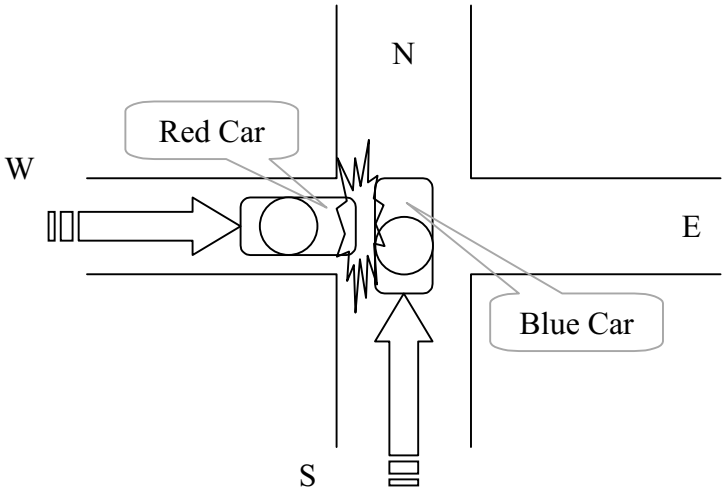
C. Max's marble exerts a smaller force on Jimmy's marble than Jimmy's marble exerts on Max's marble..

D. Jimmy's marble exerts a force on Max's marble but Max's marble doesn't exert a force on Jimmy's marble

E. Max's marble exerts a force on Jimmy's marble but Jimmy's marble doesn't exert a force on Max's marble.

F. None of the answers above describes the situation correctly.

6. A blue car and a red car both enter an intersection where the traffic light is just broken. Both cars are the **same** model except for the color. As shown in the figure below, the blue car was originally travelling from south to north and the red car was from west to east. At the moment when the red car hit the blue car on the left side, both cars are traveling at a **same** constant speed. Which of the following statement is true?



A. Red car exerts a greater force on Blue car than Blue car exerts on Red car.
B. Red car exerts a same amount of force on Blue car as Blue car exerts on Red car.
C. Blue car exerts a greater force on Red car than Red car exerts on Blue car.
D. Blue car exerts a force on Red car but Red car doesn't exert a force on Blue car.
E. Red car exerts a force on Blue car but Blue car doesn't exert a force on Red car.
F. None of the answers above describes the situation correctly.

Use the following choices to answer questions **7 and 8**.

A. John exerts a greater force on Tom than Tom exerts on John.
B. John exerts a same amount of force on Tom as Tom exerts on John.
C. Tom exerts a force on John but John doesn't exert a force on Tom.
D. Tom exerts a greater force on John than John exerts on Tom
E. John exerts a force on Tom but Tom doesn't exert a force on John.
F. None of the answers above describes the situation correctly.

7. In a soccer game, two player, John and Tom who happen to have **same weight**, are running to chase a ball that is flying close to them. John runs about **twice as fast** as Tom. Unfortunately, neither players notices the other, and they run into each other. At the time they hit, which of the above statements is true?

8. Still in that soccer game, John and Tom run into each other again. This time, Tom starts early and is running at a **constant speed**. John starts late and is **speeding up**. At the time they hit, they both happen to be running at the **same speed**. Which of the above statement is true?

9. Later in that soccer game John runs into another player, Bill, who is almost **twice as heavy** as John. This time, they are both running at the **same speed**. At the time they hit each other, which of the following statement is true?

A. John exerts a greater force on Bill than Bill exerts on John.
B. John exerts a same amount of force on Bill as Bill exerts on John.
C. Bill exerts a force on John but John doesn't exert a force on Bill.
D. Bill exerts a greater force on John than John exerts on Bill.
E. John exerts a force on Bill but Bill doesn't exert a force on John.
F. None of the answers above describes the situation correctly.

10. As shown in the figure below, an enemy aircraft drops a bomb, which is gliding down at a constant speed toward people on the ground. A missile is launched and is accelerating toward the bomb. The missile and the bomb happen to have the **same weight**. At the moment the missile hits the bomb, they both are moving at a **same speed** and neither of them explodes after the hit. Which choice describes the forces?

A. The missile exerts a greater force on the bomb than the bomb exerts on the missile.
B. The bomb exerts a greater force on the missile than the missile exerts on the bomb.
C. The missile exerts the same amount of force on the bomb as the bomb exerts on the missile.
D. The missile exerts a force on the bomb but the bomb doesn't exert a force on the missile.
E. The bomb exerts a force on the missile but the missile doesn't exert a force on the bomb.
F. None of the answers above describes the situation correctly.

Use the following choices to answer questions **11 and 12**.

A. The floor exerts a greater force on the feet than the feet exert on the floor.

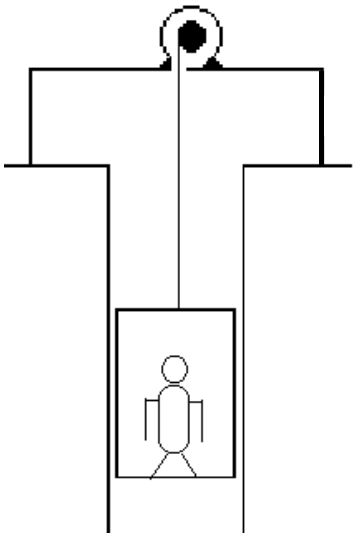
B. The feet exert a greater force on the floor than the floor exerts on the feet.

C. The feet exert a force on the floor but the floor does not exert a force on the feet.

D. The floor exerts a force on the feet but the feet do not exert a force on the floor.

E. The floor exerts the same amount of force on the feet as the feet exert on the floor.

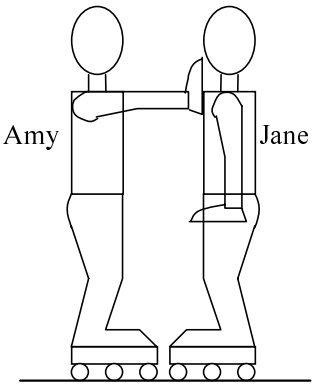
F. None of the answers above describes the situation correctly.



11. Anna is taking an elevator from fourth floor of the physics building to catch the next class in the basement. In the elevator, Anna is standing right in the middle without touching any thing else. When the elevator just **started to go down.** Which above choices describes correctly the forces between the floor of the elevator and Anna's feet?

12. After class, Anna is taking the same elevator from the basement back to fourth floor to ask for help from his physics professor. In the elevator, Anna is standing right in the middle without touching any thing else. When the elevator just started to go up. Which above choices describes correctly the forces between the floor of the elevator and Anna's feet?

13. Two students, Amy and Jane, are on very good identical roller blades facing each other. They both have a same mass of 50 kg. Amy places her hand on Jane, as shown to the right. Amy then suddenly pushes outward with her hand, causing both to move. In this situation, while Amy's hands are in contact with Jane, which choice describes the forces?



A. Jane exerts a force on Amy, but Amy doesn't exert any force on Jane.

B. Amy exerts a force on Jane, but Jane doesn't exert any force on Amy.

C. Each student exerts a force on the other, but Jane exerts the larger force.

D. Each student exerts a force on the other, but Amy exerts the larger force.

E. Each student exerts the same amount of force on the other.

F. None of these answers is correct.

14. Susan, a little girl, and her Mom are traveling to Europe. They have a lot of luggage. At the airport, Susan tries to help her Mom by handling her luggage all by herself. She has a big suitcase that weighs the same as she. So she can only push the case to move it a little bit at a time. But she keeps doing it. Each time while she is pushing the case, which choice describes the forces?

A. The case exerts a force on Susan, but Susan doesn't exert any force on the case.

B. Susan exerts a force on the case, but the case doesn't exert any force on Susan.

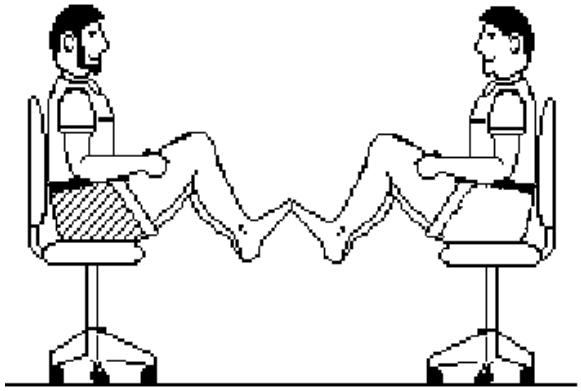
C. Both Susan and the case exert a force on the other, but the box exerts a larger force on Susan.

D. Both Susan and the case exert a force on the other, but Susan exerts a larger force on the box.

E. Both Susan and the case exert the same amount of force on the other.

F. None of these answers is correct.

15. Two students, Bob and Jay, sit in identical office chairs facing each other. Bob has a mass of 100 kg and Jay has a same mass of 70 kg. Both Bob and Jay place their feet against the other, as shown to the right. They then both suddenly push outward with their feet at the same time, causing both chairs to move. In this situation, while their feet are still in contact, which of the following choices describes the force?



A. Jay exerts a force on Bob, but Bob doesn't exert a force on Jay.

B. Bob exerts a force on Jay, but Jay doesn't exert a force on Bob.

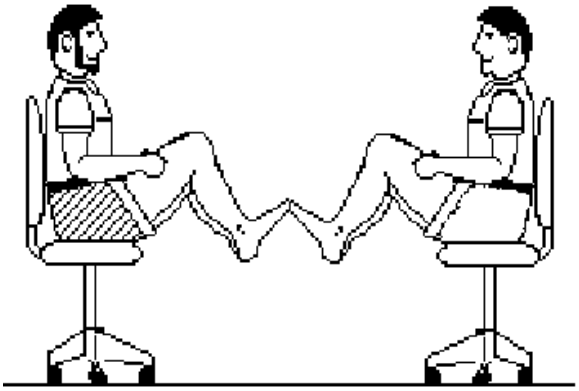
C. Each student exerts a force on the other, but Jay exerts the larger force.

D. Each student exerts a force on the other, but Bob exerts the larger force.

E. Each student exerts the same amount of force on the other.

F. None of these answers is correct.

16. Now Bob and Peter sit in identical office chairs facing each other. They all have the same mass of 100kg. Peter is a football player and is much stronger than Bob. Again, they then both suddenly push outward with their feet, causing both chairs to move. In this situation, while their feet are still in contact, which of the following choices describes the force?



A. Peter exerts a force on Bob, but Bob doesn't exert a force on Peter.

B. Bob exerts a force on Peter, but Peter doesn't exert a force on Bob.

C. Each student exerts a force on the other, but Peter exerts the larger force.

D. Each student exerts a force on the other, but Bob exerts the larger force.

E. Each student exerts the same amount of force on the other.

F. None of these answers is correct.

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