

Problem Solving and Conceptual Understanding

William J. Gerace

*Physics Department and Scientific Reasoning Research Institute
University of Massachusetts, Amherst MA 01003-4525*

A framework for thinking about knowledge and its organization is presented that can account for known expert-novice differences in knowledge storage and problem solving behavior. It is argued that interpreting any relationship between the ability to answer qualitative and quantitative questions requires a model of cognition, and that PER should seek to develop assessments that monitor component aspects of developing expertise.

Relating qualitative understanding to quantitative problem solving ability

There have been a number of efforts to correlate students' abilities to answer qualitative and quantitative questions, and research findings are not consistent. Should we find this surprising? I think not. We do not have a clear definition of "qualitative question", or, for that matter, of "quantitative question". Without well-defined categories, why should any correlation be expected? Even with good definitions and clean data, a model is required to interpret any observed correlation.

In the following I present the framework used by the UMass Physics Education Research Group to explain research-identified expert-novice differences. ([1] explains how we use this framework, along with a list of cognitive goals and instructional tactics, to develop curriculum materials.) This frame-

work helps us understand the relationship between qualitative reasoning and quantitative problem solving skills, and suggests the circumstances under which one ought to expect a correlation.

Based on this framework, we have developed a pedagogic approach we call "concept-based problem solving". (Details of the approach can be found in [2] and [3].) The framework and pedagogic approach are predicated upon three premises:

- 1) particular types of knowledge and knowledge structures are needed for proficient problem solving;
- 2) particular types of cognitive processes are required for acquiring conceptual knowledge and building useful knowledge structures; and
- 3) one can design activities that promote these desirable cognitive processes.

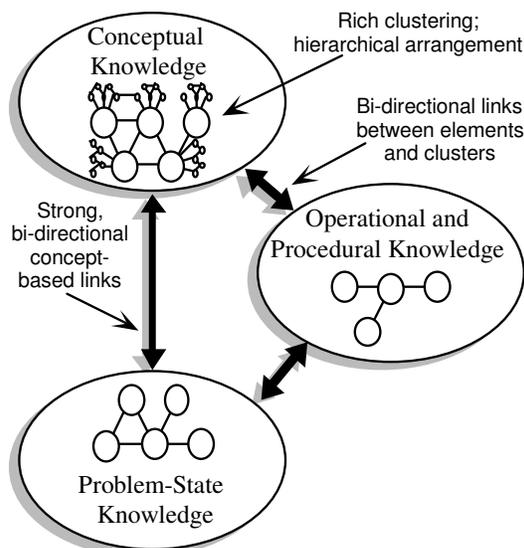


Fig. 1: Expert's knowledge store.

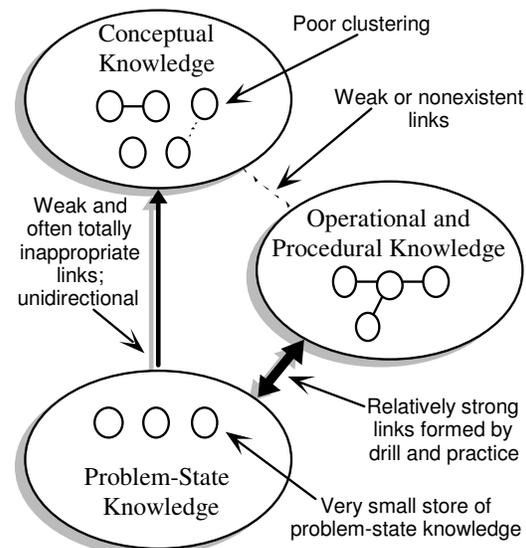


Fig. 2: Novice's knowledge store.

Expert-novice differences

Fig. 1 represents knowledge possessed by an expert. Note the structuring of the conceptual knowledge store, and the strong bi-directional linkage between conceptual knowledge and problem state knowledge created through the encoding of problems. In contrast, the novice (Fig. 2) has poor conceptual knowledge structuring, weak unidirectional links from problem state knowledge to concepts, and relatively strong practiced links between problem state knowledge and operational knowledge. Tables 1 and 2 summarize known expert-novice differences, used to construct this framework.

Strategic knowledge elements (Fig. 3) enable an expert to devise forward looking, concept-based problem solving methods. These elements (often termed *schema*) are compound elements consisting of a concept or principle, operational knowledge needed to employ the concept, and characteristics of problem situations in which the concept is useful. Strategic knowledge organizes problem-state knowledge, allowing an expert to perceive problems' "deep structure".

It seems to me that any correlation between students' performance on conceptual and quantitative questions would depend on the degree to which they are related through strategic knowledge elements (though a cor-

Expert	Novice
Conceptual knowledge impacts problem solving	Problem solving largely independent of concepts
Often performs qualitative analysis, especially when stuck	Usually manipulates equations
Uses forward-looking concept-based strategies	Uses backward-looking means-ends techniques
Has a variety of methods for getting unstuck	Cannot usually get unstuck without outside help
Is able to think about problem solving while problem solving	Problem solving uses all available mental resources
Is able to check answer using an alternative method	Often has only one way of solving problem

Table 1: Summary of major differences between experts' and novices' problem solving behavior, from problem solving studies.

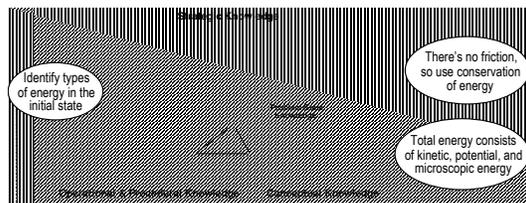


Fig. 3: Representation of an expert's strategic knowledge.

relation could be masked by differences in question representation and in the operational/procedural knowledge required).

Assessment of developing expertise

Relating students' ability to answer qualitative questions to their quantitative problem solving ability is only part of the task confronting the PER community. Why do we care about problem solving at all? We care because we believe problem solving measures students' ability to *apply* physics knowledge to accomplish an objective. But problem solving requires a complex mix of skills and experiences; measuring overall competence, while certainly of some value, does not help us directly understand and assess the cognitive structures underlying expertise or optimally structure the educational experience.

I think it is of value to occasionally step back from the outcomes oriented viewpoint that has come to dominate PER, and to consider other goals and methodologies. To this end, I will briefly present a cognitive-style research study investigating ways to assess students' knowledge store characteristics. Ideally, assessments for the individual components of the skills and experiences involved in problem solving would enable us to optimize the learning process.

The *ConMap* ("Conceptual Mapping")

Expert	Novice
Store of domain-specific knowledge	Sparse knowledge set
Knowledge richly interconnected	Knowledge mostly disconnected, amorphous
Knowledge structured hierarchically	Knowledge stored chronologically
Integrated multiple representations	Poorly formed and unrelated representations
Good recall	Poor recall

Table 2: Summary of major differences between experts' and novices' declarative knowledge characteristics, from problem solving studies.

study was our initial attempt to develop a mechanism for gauging the interconnectedness and organization of a student's knowledge store. It was motivated by observed expert-novice differences and our framework for understanding them, as described above. The research is ongoing. Our findings to date, from Ian Beatty's thesis work, are presented in [4] and [5].

The difficulties involved in developing assessment instruments for individual cognitive processes and capacities may seem daunting. I submit, however, that they are no less surmountable than those encountered in other fields of physics. As physicists, we should be devising testable *models of cognition* useful for predicting learning and performance. In a field so young, it is important to keep in mind the relationship between a model, empirical data, and the experimental instrument. Fig. 4 depicts this relationship. All three elements and their interactions must be reconsidered regularly if significant advances are to be made.

In the ConMap study, students from introductory physics courses (Mechanics and E&M) were given sets of computer-based tasks. One task involved presenting students with a prompt term and having them type ten related terms. A second was similar, but the prompt was a problem rather than a term. A third had students type an exhaustive, unplanned list of all terms from a topic like "mechanics". Participating students engaged in combinations of these tasks regularly through the semester. They also periodically constructed traditional hand-drawn concept maps. It is not possible to present much detail about the project in this limited space, so I will briefly mention three findings of particular interest.

1) These computer-based tasks appear to probe the same knowledge structures as do hand-drawn concept maps. Tables 3 and 4

prompt	mean (st.dev.)
"force"	0.54 (0.22)
"energy"	0.61 (0.19)
"momentum"	0.64 (0.18)
"force" (second session)	0.47 (0.18)
all 4 combined	0.57 (0.20)

Table 3: Fraction of "innermost" hand-drawn concept map terms appearing in term-prompted term entry response list, averaged over subjects.

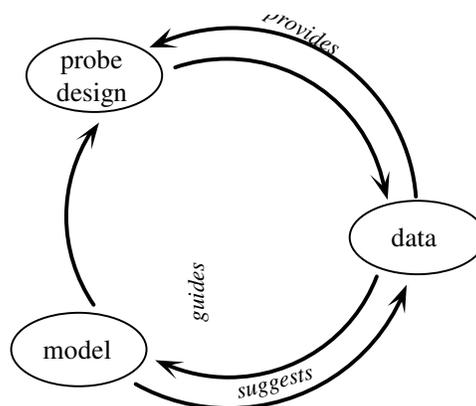


Fig. 4: Interrelationships between experimental probe, data gathered, and model constructed.

summarize the overlap of terms appearing in the first activity mentioned above with those appearing in a hand drawn concept map. On average, 57% of "innermost" terms in the map occur in the list of terms, and 82% of the list terms occur somewhere on the map.

2) The "quality" of students' response lists for the prompt term *force*, according to experts' judgment of relevance, is found to correlate with overall exam performance (see fig. 5). The exams included many non-force topics, so this is an interesting result.

3) Fig. 6 indicates that the frequency of unrelated successive terms in students' responses to a topic area prompt correlates inversely with exam score, suggesting that appropriate knowledge store structuring has a positive influence on performance.

Again, these results are intended to suggest the feasibility of developing assessment instruments for gauging "internal" measures of students' developing expertise.

Assessing Instructional Approaches

The PER community has expended considerable effort assessing the efficacy of instructional approaches for delivering con-

prompt	mean (st.dev.)
"force"	0.81 (0.20)
"energy"	0.87 (0.13)
"momentum"	0.87 (0.14)
"force" (second session)	0.72 (0.12)
all 4 combined	0.82 (0.16)

Table 4: Fraction of term-prompted term entry responses appearing on corresponding hand-drawn concept map.

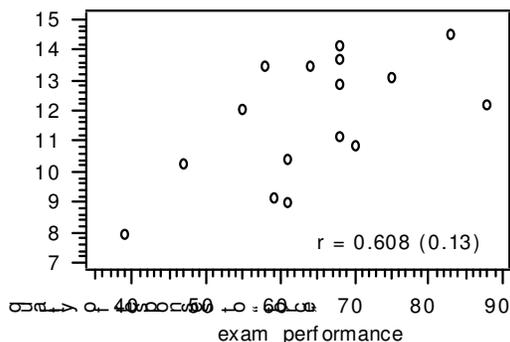


Figure 5: Term-prompted term entry list score for the prompt “force” vs. sum of raw exam scores on all four course exams (Physics 151).

ceptual knowledge, e.g. with the *Force Concept Inventory* (FCI). As with assessing the component skills of problem solving, we are confronted by an extremely complex mix of factors, many of which cannot be controlled. Although the FCI has been shown to be a valid instrument, gleanig information about instructional practices from FCI scores has proven difficult. So far, one of its most successful applications has been demonstrating the ineffectiveness of traditional teaching methodologies. The positive gains indicated by many different curricular materials and methodologies raise more questions than answers. Are these effects linear, cumulative, and persistent? Why is it apparently so easy to get conceptual gains and so difficult to improve problem solving?

For years many educators, ourselves included, have said that instruction should become more learner-centered, stress process over information, and engage students in dialogue. The progress that innovative curriculums have made in the delivery of conceptual knowledge is, I believe, largely due to their incorporation of one or more of these aspects. We must distinguish the forest from the trees and identify what common features underlie different, apparently successful instructional innovations.

In my view, we need to devote more attention to cognitive research issues and investigate other characteristics of expertise. We need to take a more cognitive approach and develop assessment measures for the component skills involved in problem solving. Ideally, studies should elucidate the relevant cognitive processes and their interaction with each other and the knowledge

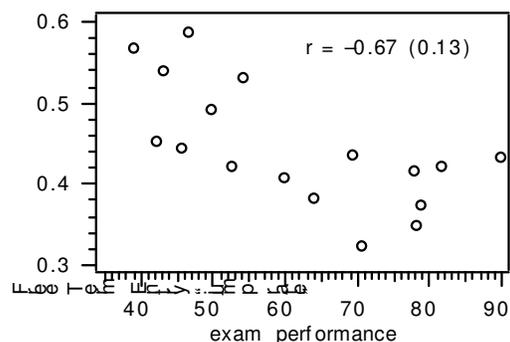


Figure 6: Fraction of Free Term Entry responses which are “jumps” (non-sequiteurs) vs. average of all four course exams (Physics 152).

store. Methods for quantifying the presence and functioning of these processes should be devised so that we can monitor and optimize the development of expertise.

References

Papers may be accessed online at:
<http://umperg.physics.umass.edu/writings/online/>

1. Dufresne, R.J., Gerace, W.J., Mestre, J.P., & Leonard, W.J. (2000) “ASK•IT/A2L: Assessing student knowledge with instructional technology.” UMPERG 2000#09-SEP#1-28pp.
2. Leonard, W.J., Gerace, W.J., & Dufresne, R.J. (1999) “Concept-based problem solving.” UMPERG 1999#12-NOV#3-18pp.
3. Leonard, W.J., Gerace, W.J., Dufresne, R.J. & Mestre, J.P. (1994 rev. 1999) “Concept-based problem solving in physics.” UMPERG 1994#08-APR#1v2-16pp.
4. Beatty, I.D. & Gerace, W.J. (2001) “Probing physics students’ conceptual knowledge structures through term association.” *American Journal of Physics* PER Supplement (in press).
5. Beatty, I.D. (2000) ConMap: Investigating New Computer-Based Approaches to Assessing Conceptual Knowledge Structure in Physics. Ph.D. Dissertation, University of Massachusetts.