Students' reasoning across contexts

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We give two examples on how students' reasoning is dependent on the context of the questions. In the first example concerning the grounding of a neutral metal in the presence of a positive external charge, we show how relations can be thought of as preserved between two related questions. In the second example, concerning the grounding of a neutral metal in the presence of a negative external charge, we show how the reasoning is dependent on the fundamental ontological reasoning students have with respect to what type of charge is capable of movement. The implication for instruction is that a set of related questions as discussed in the paper can be used to elucidate and address the fundamental limitations of students' reasoning.

Introduction

The question we study in this paper is whether students' reasoning can be understood as a set of mapping relations. We start with a hypothesis for the student reasoning and then test to what extent the predictions hold. The questions are motivated by our belief of what ontological commitments [1] students possess in the domain of electric phenomena and the possible mental models students might hold regarding such phenomena [2].

We take an approach in which we probe students' reasoning with a set of related multiple-choice questions. The set of questions are related in the sense that they can be categorized as belonging to the same conceptual domain. The questions can also be thought of as related in the sense that the variability in contextual features are kept to a minimum.

The first set of questions concern the grounding of a neutral conductor in the presence of a positive external charge. The second set of questions involves the same but in the presence of a negative external charge.

The grounded conductor problem (see figures – next page) appeared in the midterm

examination of a calculus-based electricity and magnetism course at the Ohio State University. The questions on the positive external charge and the negative external charge were given to two groups of students from the same population who received instruction from the same instructors.

A grounded conductor – positive external charge

The question was fundamentally motivated by the ontological commitment shown by students to reason in terms of attraction and repulsion of charges – perhaps due to direct associations. Furthermore, causal representations provided in the discussions of charging by induction follow a similar pattern across many instructional environments including many textbooks; namely, that the side farthest to the external charge when grounded remove the like charges to that of the external charge. We hypothesized two models based on such learning.

1. If by grounding the end farthest to that of the external charge remove the like charges to that of the external charge, then grounding the near end should remove the unlike charges. In short, the type of charge removed is dependent on where the ground connection is made. 2. The ground connection removes all charges thereby leaving the conductor neutral. The following two questions were given to test these hypotheses.

Q-1:

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.



What is th \vdots ge on the conducting rod after the ground co. \bigoplus n is removed?

A. Positive charge

B. Negative charge

C. No charge (Neutral)

Q-2:

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end closer to the charge is then connected to the ground by a conducting wire as shown below.



What is the charge on the conducting rod after the ground $\bigoplus_{\text{ction is removed}}^{\text{the charge on the conducting rod after the set of the se$

A. Positive charge B. Negative charge C. No charge (Neutral)

The response distribution and the transitions between the responses for the two questions are given in Table 1 and Table 2, respectively.

N=89	Q-1	Q-2
А	$10 \pm 6 \%$	37 ± 10 %
В	$66\pm10~\%$	$29\pm9~\%$
C	24 ± 9 %	$34 \pm 10 \%$

Table 1: Response distribution for Q-1 and Q-2.

$Q-2 \rightarrow$	А	В	С
Q-1↓			
А	4 ± 4 %	3 ± 4 %	2 ± 3 %
В	29 ± 9 %	26 ± 9 %	11 ± 7 %
С	3 ± 4 %	0 %	$20\pm8~\%$
Table 2: Trai	nsitions betw	een the respo	nses for O-1

Table 2: Transitions between the responses for Q-1 and Q-2.

We see that the selection of choice B is prominent in the context of Q-1 and drops significantly in the context of Q-2. On the other hand the selection of choice A in the context of Q-2 increases significantly. (The error is calculated assuming a binomial distribution.)

The above results support our hypothesis that students would view where the grounding is placed as a significant parameter and hence the charge that would remain on the conductor will be dependent on it. The correct reasoning is that in both cases the choice that should be selected is B.

We can expect a student who selects choice C in the context of Q-1 to also select the same in the context of Q-2 since there are no changes within the question contexts that would suggest or cue a major shift from having a "neutral model" to another.

A grounded conductor – negative external charge

An added hypothesis we formed in considering the grounding problems is that the type of external charge may influence students' reasoning. This was based on our knowledge of what students would manipulate fundamentally as the mobile charge. The mobile charges are taken to be electrons, which are negatively charged. If this is the case, how electrons are transferred from the ground to the neutral conductor can have an effect when the external charge itself is negative due to propositions such as "like charges repel." The following two questions were given with this idea in mind.

Q-3:

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.



What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge B. Negative charge

C. No charge (Neutral)

Q-4:

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.



What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge B. Negative charge C. No charge (Neutral)

The response distribution and the transitions between the responses for the two questions are given in Table 3 and Table 4, respectively.

N=86	Q-3	Q-4
Α	$76 \pm 9 \%$	$31 \pm 10 \%$
В	6 ± 5 %	$26 \pm 9 \%$
С	19 ± 9 %	$43\pm10~\%$

Table 3: Response distribution for Q-3 and Q-4.

$Q-4 \rightarrow$	А	В	С
Q-3↓			
А	23 ± 9 %	$24 \pm 9 \%$	28 ± 9 %
В	3 ± 4 %	0 %	2 ± 3 %
С	5 ± 4 %	1 ± 2 %	$13\pm7\%$

Table 4: Transitions between the responses for Q-3 and Q-4.

Comparing Q-1, Q-2 and Q-3, Q-4, there seems to be a statistically significant

difference in relation to the occurrence of choice C. That is, when the positive external charge is involved, 11% shifted to choice C in Q-2 after selecting the correct choice B in Q-1 whereas when the negative external charge is involved, 28% of the students shifted to choice C in Q-4 after selecting the correct choice A in Q-3. A contingency table can be drawn for transitions with respect to the correct choice. For example, in the context of O-1 and O-2, 23 students were consistent in selecting choice B (i.e., $- \rightarrow -$), while 26 students shifted from choice B to choice A $(- \rightarrow +)$. Ten students shifted from B to C (call it a shift to neutral or \rightarrow 0). A similar identification can be made between Q-3 and Q-4 with respect to choice A. The contingency table is given below.

N=124	$-(+) \rightarrow -(+)$	$\textbf{-}(+) \textbf{\rightarrow} \textbf{+}(\textbf{-})$	$\rightarrow 0$
+	23	26	10
(-)	20	21	24
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Table 5: Contingency table for response transitions in the presence of positive and negative external charge.

The contingency table above yields a χ^2 (2, N=124) = 6.23, p < 0.05. Thus, we may conclude that the sign of the external charge is statistically significant in determining the transitions. We see from Table 5 that the dominant factor in determining this significance is the selection of choice C.

Questions Q-3 and Q-4 were also given to the algebra-based students (N=47) and show that there is a statistically significant difference in the responses for Q-3 (χ^2 (2, N=133) = 18.97, p < 0.05) but not for Q-4 (χ^2 (2, N=133) = 4.43, p > 0.05) between the calculus- and algebra-based students. Consider the following explanation by an algebra-based student:

Q-3: [Choice A]... The object is grounded, but when the negative charge is brought close, the free electrons will be repelled and move down the wire, leaving the conducting rod positive.

Q-4: [Choice C]... The negative [external] charge would keep all the electrons from moving down the wire leaving the same amount in the rod.

This shows the fundamental reasoning in terms of electrons and the effect the external negative charge has in generating the "neutral" solution in the context of Q-4. Although explicit explanations from the calculus-based students are absent the sketches they made suggests similar reasoning.

Conclusion

We see how students can be thought of as functioning by preserving certain relation structures between questions. For example, the knowledge of polarization coupled with the removal of the like charges seems to lead most students to the correct choice in the context of Q- 1. The question Q-2 can be thought of as a novel context. In this case students seem to construct relations such that they preserve the structure of the familiar argument (e.g., if grounding one side remove positive charges then grounding the opposite side remove the (opposite) negative charges).

The fundamental reasoning involving electrons as the mobile charge has an effect on their selection of choices in the presence of the external negative charge. This problem is not present in the case of the positive charge because negative mobile charges (electrons) have an attractive force between the positive external charge or the positive charges that are separated through polarization. As such, the electrons are either attracted from the ground toward the metal rod or the positive external charge exerts enough force to make them leave to the ground at the near side.

As can be seen, most students select the correct choice in the contexts of Q-1 and Q-3. These are familiar situations. In instruction, O-2 and Q-4 can be used as an opportunity for students to expand their knowledge on the concepts and also to emphasize on the fundamental limitations in adhering to particular representations such as polarization of charges and grounding of a one particular side. This may help students realize the need for more powerful representations such as the electrostatic potential and the field. Since instruction cannot address all such issues in one single question and given the fact that we have to initialize learning with some set of representations, the questions can be used to expand the context of the questions with little variability and at the same time reveal the deeper structure necessary in understanding such phenomena.

References

[1] L. B. Resnick, Ontological commitments in learning physics. Paper presented at the meeting of Colloque International, CIRADE, Montreal (1988).

[2] P. N. Johnson-Laird, *Mental Models: Towards a Cognitive Science of Language, Inference and Consciousness,* Cambridge: Harvard University Press (1983).

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