

Considering Factors Beyond Transfer Of Conceptual Knowledge

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Abstract. One thread in education research has been to investigate whether and in what ways students “transfer” conceptual knowledge from one context to another. We argue that in understanding students’ reasoning across contexts, it can additionally be productive to attend to their *epistemological framing*. We present a case study of one student (Will), whose reasoning on two similarly structured approximation problems does not draw on pieces of conceptual knowledge across contexts in a manner that experts might view as productive. We further show that attending to Will’s epistemological framing aids our understanding of *why* he draws on different types of knowledge on the two problems.

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INTRODUCTION

Students’ abilities for transferring conceptual knowledge from one context to another are of central interest to instructors and education researchers. Both classical transfer experiments that document whether or not students apply previously learned ideas to new problems or situations [1] and more recent theories that treat transfer as the dynamic and context-sensitive activation of fine-grained pieces of knowledge [2,3] have attended primarily to the *conceptual knowledge* that is used (or not used).

Other views of transfer have argued for the value of theoretical constructs of transfer that attend to more than just application of conceptual knowledge [4]. In this paper, we will argue for the usefulness of one such construct: *epistemological framing*.

Hammer *et al.* [2] define an “epistemological frame” as a set of expectations about what knowledge one expects to use in a particular situation. For example, Hammer *et al.* describe one student who epistemologically frames solving a physics task as requiring a formal calculation, and a contrasting student who frames solving the physics task as an opportunity for intuitive sense making. Consequently, these two students bring different types of conceptual knowledge, either related to formal calculations or intuitive ideas, to bear on solving this task. So attending to students’ epistemological framing of various situations was shown to contribute to an understanding of why students apply certain conceptual knowledge in certain situations.

We support Hammer *et al.*’s argument – that attending to students’ epistemological framing can

provide explanatory power beyond attending to conceptual reasoning in transfer – with an empirical case study of one student, Will. First, attending to the conceptual knowledge he uses on two similarly structured tasks, we make the case that, in a “classical transfer” sense, Will does not apply certain pieces of his conceptual knowledge across both problems in a way that experts may view as productive. Then, we show a difference in his epistemological framings of what type of knowledge is appropriate for solving the two tasks and make the case for how this helps us to understand why he did not apply certain pieces of his knowledge to both.

We extend the work of Hammer *et al.* by applying and showing the usefulness of epistemological framing for understanding the type of empirical data typical in transfer studies: a student’s responses to two similarly structured problems.

METHODS

In the spring of 2011, 15 students participated in semi-structured interviews to investigate their reasoning about approximations on introductory physics and calculus content. Interviews consisted of several questions, during which students reasoned aloud and an interviewer asked clarifying questions. Students were able to use a calculator on these tasks.

The case study we present here is of undergraduate student Will, who had recently completed his first year of college and decided to become a mechanical engineering major. Will was selected for case study because of his clear articulation of what types of reasoning he perceived as appropriate

on the two tasks. Future work will investigate whether the findings from this case study can be used to understand other interviewed students.

The two tasks posed to Will were PENDULUM and ARCTAN, in that order, (abridged) below:

PENDULUM (a diagram of a pendulum was provided)
You have a pendulum made of a metal ball (1 kg) on a string (1 m). The approximation for the period of a pendulum for small oscillations is:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where T is the period of the pendulum, l is the length of the pendulum, and g is acceleration due to gravity. This equation only holds for small angle oscillations of the pendulum. For larger angles, the period of a pendulum can be found with the following equation:

$$T = 2\pi \sqrt{\frac{l}{g} \left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \dots \right)}$$

where θ_0 is the angle of displacement of the pendulum from vertical in radians. How big can the angle of displacement of the pendulum be before the equation for small oscillations isn't a good approximation?

ARCTAN

The Taylor series about $x = 0$ for $\arctan(x)$ is:

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$$

How big a value can x be before stopping at the second term is a bad approximation?

At the time of the interview, Will had studied Taylor series approximations, but not pendulum oscillations, in his classes.

Both tasks asked students to compare an expression representing an approximation of a quantity with a series expansion that represents that quantity's true value, and make judgments about the suitability of the approximation. Although the functional forms of the series expansions were not identical, these tasks were designed to be structurally similar to highlight ways in which students take up the concept of Taylor series differently – possibly related to the “background contexts” of oscillation as seen in physics classes, or Taylor series tasks as found in calculus texts.

Additionally, these tasks were intentionally designed to have no standard, correct answers.

Because “bad approximation” is not well defined, these tasks are designed to reveal how a student might make judgments based on contextual cues, rather than whether students can solve these tasks “correctly.”

DATA AND ANALYSIS

First, we present a summary of parts of Will's reasoning on PENDULUM and ARCTAN. Following this, we discuss some pieces of knowledge that experts might expect Will to apply from one problem to the other, but did not. Specifically, Will attempted to apply knowledge from his calculus class on ARCTAN – such as conditions for convergence of infinite series and the general formula for Taylor series expansion – but not on PENDULUM. Beyond documenting this classical “failure to transfer,” we will show how Will's epistemological framing of what constitutes appropriate reasoning on these two tasks can help us understand his approaches to them.

Summary Of Will's Reasoning On PENDULUM And ARCTAN

On PENDULUM, Will set a lower bound by deciding that $\theta_0 = 0$ did not make sense, even though the two equations for period were exactly equal:

“Well, then it's not even moving. And then ... there's no period, because it's just sitting there. Um, although, technically if theta were ... equal to zero, the two equations would equal each other, but it wouldn't be doing anything. It would just be hanging straight down. You can't really have a period when it's just sitting there not moving.”

He also set an upper bound on θ_0 by looking at the picture of the pendulum in the problem statement. From the picture, Will decided that an angle of $\pi/2$ was a reasonable upper bound, because larger pendulum swings seemed unreasonable (“I doubt it would fly all the way up here”). In comparing the two equations, he found that for the two expressions to be approximately equal, he ought to keep $(1 + \theta_0^2/16 + 11*\theta_0^4/3072)$ close to a value of 1. He plugged in his upper bound of $\pi/2$ and found that the polynomial has a value of 1.17 - “too different” from an (admitted as arbitrary) acceptable value of 1.1, based on his experiences in physics class. Will expressed a desire for a more analytical way to solve this task, because he felt that, while productive, plugging in values to find a good approximation is “not doing it right.”

On ARCTAN, Will tried to remember facts from calculus class – specifically, equations from the chapter of the book related to Taylor series. He

vaguely remembered conclusions of the various convergence tests for infinite series, writ large (e.g. the conclusions one could make regarding convergence when using the ‘ratio test’). Attempting to recall these convergence conditions, he determined that a good approximation meant that the first two terms $(x - x^3/3)$ should be between 0 and 1, but deemed this unprincipled (“I can't justify it, and I know it's probably not right ... I just remember those kind of ideas, so from there, [try] to make an educated guess”) and tried to remember the asymptotes of $\arctan(x)$ to redefine a more sensible bound. Throughout his work on this task, Will tried to remember relevant ideas from his courses, including the graph for $\arctan(x)$ and the general formula for Taylor series expansion. He stated that knowing the graph and asymptotes for $\arctan(x)$ would help him “know which answers are impossible and which answers are possible.” The interviewer provided the range of arctangent ($-\pi/2$ to $\pi/2$), and Will concluded that $-\pi/2 < (x - x^3/3) < \pi/2$, because the approximation should not go “beyond what my function is defined as.” The interviewer also provided the general formula for Taylor series expansion, and Will tried to use it to inform his decision of “good approximation.” However, he eventually decided that the general formula was not helpful for defining “good approximation.”

Comparing Will’s Different Approaches

Attending to the conceptual knowledge he brings to bear on these problems, we notice that, on ARCTAN, Will seeks conceptual knowledge from his calculus class about Taylor series, whereas he does not on PENDULUM. Experts may expect that knowledge about Taylor series from calculus class would be applied productively to PENDULUM. Instead, Will uses the physical motion of the pendulum and a heuristic sense of “good enough” on PENDULUM.

For transfer researchers, the important question is ‘what might help us understand why Will implements different sets of knowledge on these two problems?’ Further analysis may point out specific differences in the tasks that contribute to the activation of different pieces of knowledge. But while this style of analysis typically focuses on conceptual differences in how the student interprets the problems (e.g. the context of PENDULUM cuing ideas about a pendulum’s physical motion), we propose that there may *additionally* be differences in a student’s *epistemological framing* of the problems. In the case of Will, we propose that his epistemological framing of the two questions is an important piece of what influences his reasoning on these two tasks and can contribute to explaining why

he does not try to apply ideas about Taylor series from calculus class to PENDULUM.

Next, we present evidence of Will’s epistemological framing of these two tasks and show how this evidence can help us understand how his epistemological framing may support different kinds of knowledge on the two tasks.

Will’s Epistemological Frames: You Can’t Reason About The Battle Of Hastings

When discussing PENDULUM, Will articulated his belief that even if one does not know the appropriate way to solve a problem, s/he can “put something down and show logical thoughts,” and receive some partial credit, saying:

You put something down, at least you were thinking about it... Um, and it's a big difference again between this stuff and, like, history. This kind of stuff you can, even if you don't know it, you can use logic and you can, uh, make connections and, and rationalize certain things and know that they're true just by looking at what you are given. You don't need to know...a specific date or a specific event to answer a question. Like if they ask you, you know, what's the Battle of Hastings, and you don't know anything about the Battle of Hastings, you're not, you can't just be like a, there were swordsmen; they fight. You can't say that.

This quote reveals Will’s epistemological framing of what is appropriate here: the difference between PENDULUM and “what is the Battle of Hastings?” is that it is possible to “reason” about PENDULUM. Will went on to say that even if one had not learned about pendulum oscillations in physics, one could read the problem and understand what the equation means and what is being asked. From there, demonstrating some logical reasoning would earn *some* credit. On the other hand, this type of reasoning is not possible when considering factual events.

This epistemological frame that ‘logical reasoning is appropriate’ coheres with aspects of Will’s approach to PENDULUM. Starting at the very beginning of the task, Will said that he had not learned oscillations yet in physics class, but that he could read the statement of the problem and make sense of the physical system and the meaning of the variables, in order to understand the equations. Also, in determining bounds for the angle of the pendulum, Will drew on his intuitive sense of the motion of the pendulum and the physical meaning of period of a pendulum. Even though Will explicitly mentioned that he did not

believe he was “doing it right,” he further stated that a grader would give him partial credit. Unlike answering “What is the Battle of Hastings?” here Will expects that it is possible to make sense of an unfamiliar task.

When working on ARCTAN, however, Will frames the task differently. Will said that one reason he “hated these problems so much” was because he could not reason through them. While the pendulum is concrete, reasoning with Taylor series on ARCTAN is “pure mathematical reasoning.”

...it's like pure mathematical reasoning that's, like, not normal reasoning. It's, you think about it a different way. You can't just think like ... as a person, like, oh yea, [a] pendulum swings to a certain point, then this happens. [Instead] you have to think about it in terms of infinity and what happens when you go to infinity. Humans don't think like that naturally, so you have to learn it.

So, Will feels that you cannot reason through tasks like ARCTAN intuitively. In this moment, even though the concepts in the task are similar to those in PENDULUM, Will described calculus as non-intuitive reasoning where one instead must defer to the types of unnatural reasoning learned in class. When the interviewer asked if he could have solved this question if he had never taken calculus before, Will responded: “Absolutely not. I would have looked at that and been like ... I wouldn't even know what to do.”

This epistemological framing of ‘relying on facts from class’ also coheres with Will’s approach to ARCTAN. When determining the bounds on a good approximation, Will tried to recall facts and equations from his calculus class, including the asymptotes for $\arctan(x)$, conclusions of various convergence tests, and the general formula for the Taylor series. Multiple times during his problem solving on ARCTAN, he attempted to remember equations and other facts “from that chapter” that he thought were relevant to the task. In contrast to his reasoning on PENDULUM, Will’s approach to the problem here is more aligned with the ways that he describes answering a question in history: he felt that he could not reason about ARCTAN, so he tried to recall facts from class.

We claim that Will’s different epistemological framings of the two tasks contribute to an understanding of why he seeks to apply the Taylor series ideas on ARCTAN and not on PENDULUM. Will applies ideas from calculus class to ARCTAN because he frames the task as requiring unintuitive knowledge that must be recalled. Likewise, Will does not seek out these ideas on PENDULUM, because he frames the task as a venue for using intuitive and logical reasoning, and he finds the “pure mathematical reasoning” from calculus class unintuitive.

DISCUSSION

Understanding the phenomenon of when and how students apply knowledge learned in one context to another context is of central interest to transfer research. Yet, attending only to similarities or differences in the substance of students’ conceptual reasoning may not reveal other explanatory factors, such as their epistemological framing. Using Will, we have shown the usefulness of epistemological framing for understanding of why he seeks to use different conceptual knowledge on these tasks.

This focus on epistemological framing suggests different implications for instruction. For example, the case of Will suggests that in problem solving, we should attend not just to the conceptual knowledge that students use, but also to their epistemological framing.

Specifically, a classical transfer perspective might prescribe instruction targeting conceptual knowledge – for example, explicitly highlighting the conceptual similarity between the structure of the two problems, and how certain pieces of conceptual knowledge can be applied productively to both problems. On the other hand, previous studies show that helping students shift their epistemological framing of a situation can also cause a corresponding shift in their conceptual reasoning [2,5]. Therefore, for Will, another possible intervention might be to induce a shift in his epistemological framing of the tasks, to be more closely aligned with the type of knowledge and reasoning we would like him apply on these tasks.

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