APPENDIX C: TUTORIAL SEQUENCE *CONSERVATION OF ANGULAR MOMENTUM*

- Pretest

- Part 1 of tutorial

- Part 2 of tutorial (version A)

- Part 2 of tutorial (version B)

- Part 2 of tutorial (version C)

- Handout

- Homework

Pretest for tutorial sequence *Conservation of Angular Momentum* (page 1 of 2)

PRETEST: CONSERVATION OF
ANGULAR MOMENTUM   Name:_____

1. A small puck is moving across the surface of a large level air table. The puck moves with constant velocity v_0 to the right without spinning.

   TOP VIEW

   Two locations are marked on the table: point A and point B. The line containing points A and B is parallel to the path of the puck.

   <u>All questions below refer to the instant shown in the figure at right</u>

   a. Is the magnitude of the angular momentum of the puck with respect to point A ($|\vec{L}_{puckA}|$) *greater than*, *less than*, or *equal to* the magnitude of the angular momentum of the puck with respect to point B ($|\vec{L}_{puckB}|$)? Explain.

   b. Describe the direction of the angular momentum of the puck with respect to point B ($\vec{L}_{puckB}$). If this quantity is zero, state that explicitly. Explain.

   c. State whether each of the following quantities is *increasing*, *decreasing*, or *remaining constant*. Explain.

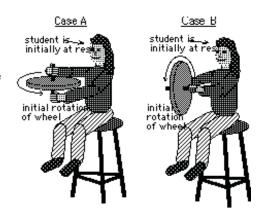   • $|\vec{L}_{puckA}|$ • $|\vec{L}_{puckB}|$

Pretest for tutorial sequence *Conservation of Angular Momentum* (page 2 of 2)

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2. A student sits at rest on a stool. The seat of the stool can rotate without friction. The student is handed a bicycle wheel that is *already spinning*. The wheel spins *without friction*.

In Case A, the axle of the wheel is vertical, and the spins counter-clockwise (when viewed from above).

In Case B, the axle of the wheel is horizontal, and the top of the wheel is moving away from the student.

In both cases, the student touches only the handles of the wheel.

Case A

Case B

student is initially at rest

student is initially at rest

initial rotation of wheel

initial rotation of wheel

a. After the student is handed the wheel in <u>Case A</u>, will she begin to rotate *clockwise* (when viewed from above), *counterclockwise*, or *stay at rest*? Explain your reasoning.

b. After the student is handed the wheel in Case B, will she begin to rotate *clockwise* (when viewed from above), *counterclockwise*, or *stay at rest*? Explain your reasoning.

Current version of Part 1 of tutorial *Conservation of Angular Momentum* (page 1 of 2)

CONSERVATION OF ANGULAR MOMENTUM

I. Angular momentum of a spinning wheel

A student sits at rest on a stool that can rotate without friction. In each of the following separate cases, the student is handed a bicycle wheel that is *already spinning*. The axle of the wheel is also low-friction.

A. For each case, *predict* whether the student will begin to rotate *counterclockwise* (when viewed from above), *clockwise*, or *stay at rest*. Explain your reasoning in each case.

Case A:
The center of the wheel is stationary. The axle of the wheel is vertical. The wheel spins counterclockwise (when viewed from above).
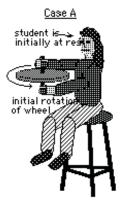
Case B:
The center of the wheel is stationary. The axle of the wheel is horizontal. The wheel spins so that the top of the wheel moves away from the student.

Check your predictions by observing the demonstration. What was the resulting motion of the student in each case?
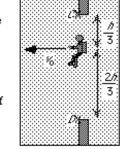
- Case A
- Case B

Just as an object moving through space has linear momentum $\vec{p}$, we say that a spinning object has *angular* momentum $\vec{L}$. It is customary to determine the direction of $\vec{L}$ for a spinning object by using the *right hand rule*. When the fingers of the right hand curl in the direction of the spinning motion, the direction of $\vec{L}$ is the same as the direction in which the thumb points.

B. For each case described above, draw a vector next to the figure to represent the angular momentum of the wheel at the instant shown.

C. On the basis of your observations thus far:

- Does a wheel that is spinning and held in place appear to exert a force on its support in the direction of its angular momentum?

- Does any part of the wheel move in the direction of the angular momentum?

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Version A of Part 2 of tutorial *Conservation of Angular Momentum* (page 1 of 2)

Note: For the following section, use the convention that $\otimes$ indicates a vector pointing *into* the page and $\odot$ indicates a vector pointing *out of* the page.

III. Angular momentum of an object moving with constant velocity

An astronaut moves to the left with constant velocity v_o, through a large doorway in a space station, as shown at right. Points C and D mark the top and bottom of the doorway, respectively, and are a distance h apart. At time $t = 0$, the center of mass of the astronaut passes directly below point C, at a distance $h/3$. The astronaut does not spin.

A. The diagram below shows the astronaut at two instants, $t = 0$, and some time later. A gray line shows the path of the center of mass of the astronaut.

1. In both boxes below, draw:

   - $\vec{r}_{aC}$ (the position vector of the astronaut with respect to point C)

   - $\vec{p}$ (the linear momentum vector of the astronaut)

2. For $t > 0$, draw $\vec{r}_\perp$ and $\vec{r}_\parallel$, the vector components of $\vec{r}_{aC}$ that are perpendicular and parallel to the linear momentum vector $\vec{p}$, respectively.

3. Write a vector equation relating $\vec{r}_{aC}$, $\vec{r}_\parallel$, and $\vec{r}_\perp$. (The Pythagorean theorem is *not* a vector equation.)

4. The angular momentum of the astronaut with respect to point C is defined by $\vec{l}_{aC} = \vec{r}_{aC} \times \vec{p}$. Replace $\vec{r}_{aC}$ in this definition by using the relationship from question 3, and explain why the definition can be simplified to $\vec{l}_{aC} = \vec{r}_\perp \times \vec{p}$.

5. What is the direction of the angular momentum of the astronaut with respect to point C? Explain.

B. Imagine that you are standing in one place, watching a friend walk by. You make sure that as your friend moves, you are always directly facing him/her. First, your friend walks in a path shaped like a semi-circle, with you at the center. Then, from the same starting place, your friend walks by with a constant velocity (in a straight line).

1. Describe how *your* motion in watching your friend is similar in the two cases.

Conservation of angular momentum

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2. Use your answer to the previous question to explain why it makes sense that $\dot{l}_{aC}$ is *not* zero (even though the astronaut is neither spinning nor moving in a curved path).

C. Does the angular momentum of the astronaut with respect to point C ($\dot{l}_{aC}$) change with time? Explain.

D. How does the angular momentum of the astronaut about point D ($\dot{l}_{aD}$) compare to the angular momentum of the astronaut about point C ($\dot{l}_{aC}$):

- in direction?   • in magnitude?

Would your answers to the above questions change if you were to determine the direction of angular momentum with your left hand instead of your right hand? Explain.

(Keep in mind that the convention is to determine the direction of angular momentum by using the right hand.)

E. Two students discuss the angular momentum of the astronaut:

Student 1: *"The angular momentum of a system does not change when there is zero net torque on the system. There is clearly no torque at all on the astronaut, so the angular momentum must stay constant as the astronaut floats by."*

Student 2: *"The angular momentum of the astronaut is not conserved because the angular momentum with respect to point C is not the same as the angular momentum with respect to point D."*

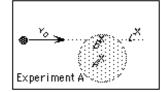With which student(s), if either, do you agree? Explain.

F. Find a reference point that is stationary with respect to the space station and for which the angular momentum of the astronaut is zero. Explain the reasoning you used to find the point.

Does the angular momentum of the astronaut with respect to this point change over time?

G. Generalize from your answers in section III to answer the following questions. For an object that is moving with *constant velocity* (and not spinning):

- Do the magnitude and direction of the angular momentum depend on the reference point?

- Does an object moving with constant velocity have constant angular momentum?

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Version B of Part 2 of tutorial *Conservation of Angular Momentum* (page 2 of 3)

Conservation of angular momentum

IV. Angular momentum of an object moving with constant velocity

The rod in the previous experiments is replaced by a disk. In three different experiments, the center of the disk is placed at different starting locations: points A, B, and C, which are marked *on the table*. In all of these experiments, the puck has the same initial velocity and moves on the path shown.

The puck sticks to the disk in all cases.

A. In the first experiment (Experiment A), the disk is placed with its center on top of point A, as shown.

   Consider the following discussion between two students:

   Student 1: *"The system must somehow have angular momentum before the collision. After the collision, the puck and disk will spin, so there is clearly some angular momentum. The angular momentum had to come from somewhere."*

   Student 2: *"That makes sense, but how there could be initial angular momentum? There's nothing angular about an object moving in a straight line."*

   With which student, if either, do you agree? Explain.

In order to account for the rotational motion of the system after the collision in a way that is consistent with angular momentum conservation, we will make the following assumptions:
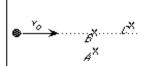
- The puck somehow *does* have angular momentum before the collision.

- The manner in which the system spins *after* the collision can be used as an indicator of both the magnitude and direction of the angular momentum of the puck *before* the collision.

For instance, if the manner in which the system spins depends on whether the disk starts at point A or point B, we say that the angular momentum of the puck *with respect to* point A is different from the angular momentum of the puck *with respect to* point B.

B. In Experiment B, the center of the disk is placed on top of point B. In Experiment C, the disk is placed on top of point C.

   Use the assumptions listed above to compare the angular momentum of the puck with respect to points A, B, and C:

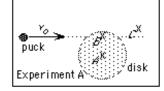   - in direction

   - in magnitude

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Conservation of angular momentum

IV. Angular momentum of an object moving with constant velocity

The rod in the previous experiments is replaced by a disk. In three different experiments, the center of the disk is placed at different starting locations: points A, B, and C, which are marked *on the table*. In all of these experiments, the puck has the same initial velocity and moves on the path shown.

The puck sticks to the disk in all cases.

A. In the first experiment (Experiment A), the disk is placed with its center on top of point A, as shown.

Consider the following discussion between two students:

Student 1: *"After the collision, the puck and disk will spin, so there must be some final angular momentum. If the system has angular momentum after the collision, it must have it before the collision, since angular momentum is conserved."*

Student 2: *"That makes sense, but how could there be initial angular momentum? There's nothing angular about an object moving in a straight line."*

With which student, if either, do you agree? Explain.

In order to account for the rotational motion of the system after the collision in a way that is consistent with angular momentum conservation, we will make the following assumptions:
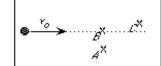
- The puck somehow *does* have constant angular momentum before the collision.

- The manner in which the system spins *after* the collision can be used as an indicator of both the magnitude and direction of the angular momentum of the puck *before* the collision.

For instance, if the manner in which the puck/disk system spins depends on whether the disk starts at point A or point B, we say that the angular momentum of the puck *with respect to* point A is different from the angular momentum of the puck *with respect to* point B.

B. In Experiment B, the center of the disk is placed on top of point B. In Experiment C, the disk is placed on top of point C.

Use the assumptions listed above to compare the angular momentum of the puck with respect to points A, B, and C:

- in direction

- in magnitude

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Handout for tutorial *Conservation of Angular Momentum*

CONSERVATION OF ANGULAR MOMENTUM: HANDOUT

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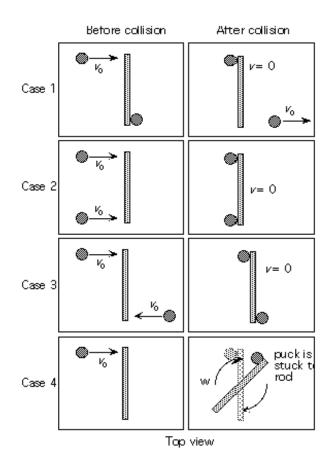
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Tutorial homework for *Conservation of Angular Momentum* (page 2 of 2)

CONSERVATION OF ANGULAR MOMENTUM

2. The four cases depicted below involve collision experiments between one or two identical pucks and a rod on a level, low-friction air table. The collisions are shown from a top-view perspective. (If any linear velocity or angular velocity is not labeled, assume it is zero. If distances appear to be equal, assume they are equal.)

For each case:

a. Indicate the direction of the total angular momentum of the pucks-and-rod system, with respect to the center of the rod, both before and after the collision.

b. Indicate the direction of the total linear momentum of the pucks-and-rod system, both before and after the collision.

c. On the basis of your answers to parts a and b, state whether the proposed process *could occur* or *could not occur*. If the process could not occur, state whether it violates (1) the principle of linear momentum conservation, (2) the principle of angular momentum conservation, or both.

Top view