

A Conceptual Approach to Physics Problem Solving

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Abstract. Students in introductory physics courses treat problem solving as an exercise in manipulating equations, symbols, and quantities with the goal of obtaining the correct answer. Although this approach is efficient for getting answers, it is far from optimal for learning how conceptual knowledge is applied in the problem-solving process. The goal of this study is to refine and evaluate an approach that encourages students to begin by writing a strategic analysis of a problem based on principles and procedures, and then to follow with a documented problem solution that exhibits, side-by-side, how concepts and equations go together in a solution. We will discuss the implementation and effectiveness of this approach in four local high school classrooms.

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INTRODUCTION

Two important goals of introductory physics courses are that students will (1) understand basic physics concepts and (2) develop problem-solving skills [1,2]. These two objectives are often treated separately on diagnostic tests or course exams, and little attention is given to their integration - the role that conceptual knowledge plays in problem solving.

In the current study, we refined an instructional intervention called Conceptual Problem Solving (CPS). CPS encourages students to perform a strategic analysis of a problem based on principles and procedures before writing down equations. The approach was implemented in first semester mechanics classes at four high schools in the fall of 2009. Each school had a control class (either the same teacher or a different teacher) in which problem solving was taught using traditional methods. In the sections that follow, we give a brief overview of the background research for Conceptual Problem Solving, describe the components of CPS in greater detail, and present information about its implementation and assessment in the four high schools.

BACKGROUND

For physicists, problem solving is fundamentally concept-driven. Their knowledge is organized around

a few big ideas (e.g. conservation laws) that can be applied in a variety of situations, and they consider underlying principles when deciding on a solution strategy [3-5]. In contrast, beginning students often focus on superficial objects and quantities in the problem statement, and they approach problems by searching for equations that match those quantities. It has been well documented that students can generate answers to standard physics problems without a strong understanding of basic concepts [6].

In addition, the structure of many courses is to address and test each physics topic in isolation, devoting little (if any) class time to the associations among major ideas or an overall synthesis of topics. As a result, students resort to memorizing equations as they need them and retain very little physics knowledge after the course.

Conceptual approaches to problem solving seek to address these issues by helping students integrate knowledge of basic physics principles with problem solving. A previous study by Dufresne, Gerace, Hardiman, and Mestre [7] found that students who analyzed problems using a menu-driven computer tool in which they identified applicable principles for problems performed more expert-like problem similarity judgments and improved problem solving.

Another study by Leonard, Dufresne, and Mestre [8] found that students who were required to write qualitative strategies before solving problems were

better at selecting which major principles apply to problems. In that study, a strategy was a written paragraph consisting of the major *Principle*, a *Justification* for why the principle is appropriate, and a general *Procedure* for applying the principle. In the following section, we explain how principle selection and strategy-writing have been adapted for the current CPS approach.

CONCEPTUAL PROBLEM SOLVING

The Conceptual Problem Solving approach is intended to be easy to implement because it does not require any major changes to the curriculum. Instructors who adopt the approach follow a particular framework when teaching problem solving in their class. The CPS approach differs from other methods of teaching problem solving in that its emphasis is on describing major concepts or principles and when they are applicable, and not necessarily on following particular “steps” while executing the solution. The main components of the current CPS approach include strategy writing and two-column solutions.

Strategy and Two-Column Solution

Whereas the “strategy-writing” described above [8] follows the structure of Principle-Justification-Procedure, the revised CPS approach uses Principle-Justification-Plan where the third step involves a numbered plan sequence outlining specifically how the principle will be applied to the problem. In addition, the strategy is followed by a solution formatted as two columns: a left column that restates the plan step, and a right column that shows mathematical equations.

The authors developed a bank of sample problems, strategies, and two-column solutions the teachers could choose from. There were typically 8-10 problems written for each physics topic. Teachers also had the option to use their own materials but structure them in the CPS format. The following text is an example of a problem with the corresponding written strategy, and Fig. 1 is a sample two-column solution.

Problem: A skateboarder enters a curved ramp moving horizontally with a speed of 6.5 m/s, and leaves the ramp moving vertically with a speed of 4.1 m/s. The skateboarder and the skateboard have a combined mass of 55 kg. Find the height of the ramp, assuming no energy loss to frictional forces.

Principle: Conservation of energy: the total mechanical energy (sum of kinetic and potential energies) of an isolated system is the same in the initial and final states.

Justification: Mechanical energy is conserved if there are no non-conservative forces that do net work

on the system. The normal force exerted on the skateboarder is a non-conservative force, but the work that the normal force does is zero because its direction is always perpendicular to the displacement. The gravitational force is conservative (it is already included in the potential energy term), and we are ignoring non-conservative frictional forces. Therefore, mechanical energy is conserved.

Plan: 1. Draw a picture and assign symbols for quantities in the problem. Choose a coordinate system. 2. Write an equation for conservation of mechanical energy. Expand the equation to include the initial and final kinetic and potential energy terms. 3. Solve for the height of the ramp. Substitute values to get an answer.

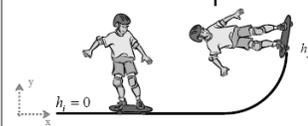
Plan Step	Equation(s) used in step
1. Draw a picture and assign symbols for quantities in the problem. Choose a coordinate system.	<p>Initial state: $v_i = 6.5 \text{ m/s}$</p> <p>Final state: $v_f = 4.1 \text{ m/s}$</p>  <p>$m = 55 \text{ kg}$ Mass of the skateboarder and skateboard combined</p> <p>$v_i = 6.5 \text{ m/s}$ Initial speed of skateboarder</p> <p>$v_f = 4.1 \text{ m/s}$ Final speed of skateboarder</p> <p>$h_i = 0 \text{ m}$ Initial height of skateboarder</p> <p>h_f Height of the ramp (final height of the skateboarder)</p>
2. Write an equation for conservation of mechanical energy. Expand this equation to include the initial and final kinetic and potential energy terms.	$\Delta E = 0 \Rightarrow E_i = E_f$ $K_i + U_i = K_f + U_f$ $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$ $\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgh_f$
3. Solve for the height of the ramp. Substitute values to get an answer.	$mgh_f = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ $h_f = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{mg}$ $h_f = \frac{\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2}{g}$ $= \frac{\frac{1}{2}(4.1 \text{ m/s})^2 - \frac{1}{2}(6.5 \text{ m/s})^2}{(9.8 \text{ m/s}^2)}$ $= 1.3 \text{ m}$

FIGURE 1. Example two-column solution.

Assessment

The CPS and control classes were evaluated using an assortment of conceptual and problem solving tests at the end of the fall term. In addition, the CPS classes were observed periodically by researchers and each CPS teacher participated in a debriefing session to provide a first-hand account of their impressions and experiences teaching with the approach.

We chose to evaluate the intervention with several different measures of conceptual understanding and problem solving. Five types of tests were administered, including:

- *Categorization*: 3-problem categorization tasks [4]
- *Conceptual questions*: multiple choice
- *Finding errors*: shown a worked-out solution that includes a physics error and asked to identify and describe the error in writing
- *Equation instantiation*: shown a problem and worked solution in symbolic form; asked to assign /match the appropriate values for each quantity in the final expression
- *Problem solving*: 3-5 free response problems

Not every school took every test due to time constraints and differences in topic coverage. The time allotted for each test ranged from 15-25 minutes.

HIGH SCHOOL PARTICIPANTS

High school teachers who agreed to participate in the study met briefly with faculty researchers during the previous summer to discuss the approach and guidelines for its implementation. All of the physics classes were selected to be non-advanced algebra-based courses (for example, advanced placement courses were avoided due to their inflexible schedule). During the fall, the teachers were provided with various materials: example problems using the CPS manipulation, a poster to be used to illustrate the structure of mechanics topics, and tests to be given at the end. The first two authors observed each teacher at least once during each physics topic (3-4 times) and maintained weekly e-mail communication.

The CPS teachers reported that they devoted approximately two class periods to the approach for each physics topic: motion with constant acceleration, Newton's laws, momentum, and energy (8 times). In general they would first model how to write a good strategy in front of the whole class, and then asked students to try it on their own or with other students while providing in-class assistance and feedback. Remaining class periods were spent on activities such as lecture, laboratories, and exams.

Before providing a sample of the results at the four schools (A-D), we address two important issues that affect their interpretation: school/teacher differences and obtaining consent. First, the student populations and curricula differ substantially across the schools so it is difficult to combine inter-school data. There were also some differences between the control and treatment groups within a single school, despite comparing the same course. For example, at school D the CPS teacher was much less experienced at teaching physics than the control teacher. In addition, there were some issues that arose in conjunction with the assessment administration process. In two of the four schools (C and D), teachers of the control classes

were reluctant to spend class time on the assessments. As an alternative a shorter out-of-class assessment was scheduled but, because it was voluntary, lost the clean interpretation of having all the students included. Since at these schools only three of the five tests were administered, we have different amounts of data from different schools. Secondly, in some schools the return rate for parental consent and student assent forms was low. The four schools are described below with a brief summary of the test results at each school. For mean score differences that are statistically significant, the p-values are provided in parentheses.

Implementation and Results

School A is a local rural high school with an experienced physics teacher. The school population is 95.3% white, 12% low income, and 68.8% of students meet state standards for AYP (Adequate Yearly Progress). This was the teacher's third year participating in the project, where his first year was used as a control class. The teacher implemented CPS with his own problems and also implemented synthesizing lectures modeled after a structure of mechanics diagram. Due to low enrollment this year, he only had 14 students in the class who returned consent forms (compared to 23 in the previous control class), but there were consistently higher mean scores for the CPS class over the control, though not always significant. The equation instantiation test showed a 12% advantage and problem solving showed a 10% difference. The categorization overall showed only a 2% difference and the finding errors test scores were too low for both groups to make a meaningful comparison. Most importantly, the conceptual questions showed a strong significant advantage for the CPS group of 10% ($p < .05$).

School B is a rural high school in which the physics teacher (trained as a chemist) taught two sections of the same class, one using CPS and one traditional. This was only his second time teaching physics. The school population is quite different from school A: 57.9% white, 53.2% low income, and 47.2% of students meet AYP standards. The teacher typically implemented the CPS approach (particularly the strategy-writing) word-for-word from the example materials and sometimes students worked on problems with a partner. In the control section the teacher was observed to use the same physics problem statements but followed a different framework when teaching problem solving: 1. write down what values are given and what you are trying to find, 2. choose equation(s), and 3. solve. Although the CPS group showed advantages on the assessments, a few difficulties may have attenuated the results. The classes were small (15

and 14 students) and did not have a great return rate on consent forms so we were left with only 17 students across the two classes. Also, because of his non-physics background and limited teaching experience, he had some difficulty implementing CPS. However, the results were quite impressive. Although there was no difference in equation instantiation or finding errors (very low scores), there were higher scores on categorization (11% advantage) and problem solving (15% advantage). Most impressively, the conceptual questions showed a large 20% difference ($p < .005$).

School C is a local suburban high school. The school is 55.8% white, 31.5% low income, and 60.0% of students meet AYP standards. The teacher made extensive use of cooperative group problem solving with the CPS approach. The control group here had some difficulties: the control teacher teaches several physics classes but was not helpful in securing consent forms or providing test time. Even so, we can point to two indications of success. First, the CPS students did better on all three tests taken: equation instantiation (6% difference), conceptual questions (11% difference), and most impressively, problem solving (16% difference, $p < .01$). Secondly, he loved the way of teaching and felt his students' conversations and questions were at a much more principle-based level (not equation-focused).

School D is another local suburban high school with a diverse student population and a wide variety of ability levels. The school is 48% white, 51.6% low income, and 49.7% of students meet or exceed AYP standards. The teacher has training in General Science Secondary Education but does not hold a physics-specific degree. It was his 4th time teaching this particular course. Here we had the most difficulty with the implementation and control group—the other teacher was much more senior and had some control over this teacher's class plans plus was unwilling to devote class time for assessment. There were no measurable differences between the CPS and control classes (1-3% for each of the tests). Overall, the CPS teacher was quite positive about the experience. The strategy and two-column solutions were graded on homework, quizzes, and tests. He liked that requiring students to write out their reasoning processes allowed him to diagnose student difficulties that were not always evident in more traditional problem solving instruction. However, he reported struggling with the amount of class time it took to go through a strategy and two-column solution in depth.

We recently spent 1-2 hours with each teacher talking about the intervention. We are still processing the information, but two issues arose consistently. First, they all liked teaching this way. Although not all of the teachers are teaching physics next year, all of them said they will continue to take a conceptual,

principle-focused approach in their future teaching. Second, most had suggestions for what might have made this work better. One issue that arose consistently is that the procedure assumes a level of understanding of the field that may be optimistic for high school students. They felt that a different way of teaching at the beginning of the year to help students understand this better might have led to even larger effects. One possible modification is to give students practice identifying principles for problems without solving them, or asking students to reflect on the principles they used after solving a problem rather than identifying principle(s) in the first step. These ideas will be explored in a revised implementation of the CPS approach at School B in the fall of 2010.

DISCUSSION

Given that the CPS approach was explicitly modeled only 6-8 times during the semester (when problem solving was being covered) and that the style of implementation varied across teachers, there was generally a small but consistent advantage to the Conceptual Problem Solving classes across all the schools. In some cases the mean test score differences between the treatment and control classes at a school were statistically significant. Overall, these results support the potential usefulness of strategy-writing and two-column solutions in high school physics classes.

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